(C) The Argument from (Natural) Numbers
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Introduction

The preface to Alvin Plantinga's "Two Dozen (or so) Theistic Arguments" warns that as the arguments are set out there they "aren't really good arguments; they are merely argument sketches, or maybe only pointers to good arguments. They await that loving development to become genuinely good" (2007a, 203). I'm going to give one of these arguments—the argument from numbers—some loving development, but I do not know whether the argument will become genuinely good. I don't know whether there's a genuinely good argument to be had.

Anyone who has been rightly taught—that is, anyone who has read van Inwagen—will know that there are virtually no good arguments for substantive philosophical conclusions (see van Inwagen 2006, Lecture 3). The problem, in part, is that it's hard to know what makes for a good argument. Plantinga implies that van Inwagen's own standards for a good argument are, at least in some cases, *too* liberal (see Plantinga 2007a, 207–208; so far as the success of our argument is concerned, they'd better be too strict). I'll be keeping in mind the numbers: how much of an audience Plantinga's argument will appeal to.

The Argument from Numbers

Alvin Plantinga sketches the following argument for the existence of God:

> *(C) The Argument from (Natural) Numbers.* (I once heard Tony Kenny attribute a particularly elegant version of this argument to Bob Adams.) It also seems plausible to think of numbers as dependent upon or even constituted by intellectual activity; indeed, students always seem to think of them as "ideas" or "concepts," as dependent, somehow, upon our intellectual activity. So if there were no minds, there would be no numbers. (According to Kronecker, God made the natural numbers and man made the rest—not quite right if the argument from sets is correct.) But again, there are too many of them for them to arise as a result of human intellectual activity. We should therefore think of them as among God's ideas. Perhaps, as Christopher Menzel suggests (special issue of *Faith and Philosophy*), they are properties of equinumerous sets, where properties are God's concepts. (2007, 213)

The argument is limited both in its conclusion and in its appeal. It's limited in its conclusion since it points only to the existence of infinite intellectual activity. As it stands, the argument doesn't point toward infinite power, infinite goodness or any other divine attributes. It doesn't even show that that the intellectual activity is that of a single intellect rather than spread out among two or more intellects: maybe there are infinitely many, infinitely puny intellects each devoted to thinking just of its own special number.

It's limited in appeal since it depends on a very specific view about the nature of numbers as dependent on intellectual activity. When prompted, students do tend to describe numbers as

"ideas" or "concepts." But then students aren't usually prompted and tend not to think about such things. I went through years of math class without stopping at all to ask what kind of things numbers are, if they are so much as things at all. And those most devoted to such questions tend not to think that numbers are ideas, though maybe that has something to do with their neglecting the divine alternative. The argument will have a quite narrow *thinking* audience.

Even so, demonstrating any divine attribute would be quite something. But it would be better if the argument could show a little more. Showing more usually requires assuming more, which means more that can be disputed—which means an even narrower audience. Ambition is traded for audience, or audience for ambition. Still, the trade might be worth it. I'll try to develop the argument so that it shows more about the divine attributes, though my ultimate conclusion will be agnostic about the argument from numbers.

But now for something not entirely different. Immediately after presenting the argument from numbers, Plantinga mentions a similar argument about properties:

> There is also a similar argument re *properties*. Properties seem very similar to *concepts*. (Is there really a difference between thinking of the things that fall under the concept horse and considering the things that have the property of being a horse?) In fact many have found it natural to think of properties as reified concepts. But again, there are properties, one wants to say, that have never been entertained by any human being; and it also seems wrong to think that properties do not exist before human beings conceive them. But then (with respect to these considerations) it seems likely that properties are the concepts of an unlimited mind: a divine mind. (Ibid)

This isn't at all as natural to me as to those many others. The bar isn't very high. I don't have any inclination to think of properties as concepts. The features of things are out there in the world with those things. Horsiness is in the horse, not in our minds. That's why there can be properties we've never thought of and that existed long before us. I don't know what to say about properties that have never been instantiated, though. I'm not so confident that there are any. In any case, if horsiness isn't a concept, then they aren't concepts either.

The argument from properties also faces a bootstrapping objection: while properties would depend on God, God would seem to depend upon properties, such as omnipotence and omniscience; if there are no such properties, then there can be no God. This threatens to impose an impossible circle of dependence (for a possible answer to the problem see Menzel 2016; and for Plantinga's own view on the relationship between God and his properties see Plantinga 1980). The argument from numbers might face a similar objection: Numbers would depend upon God, even while God would depend on numbers, such as the number one; God is one, an absolute unity and unique. This threatens to impose an impossible circle.

I'll drop this problem, though I will address another vicious circle of dependence—in another argument to be had from properties, an argument that needn't take properties to be concepts. I can't promise it will appeal to a wider audience, since it assumes some principles of its own. I think these principles have more to be said for them than the idea that properties are concepts. The bar isn't very high. If the audience is widened, then it's just by relinquishing the assumption that properties are concepts. Without that, there's less at my disposal to work with in

one way—which means reaching less interesting conclusions. Ambition is traded for audience, or audience for ambition.

The Argument from Properties

The other argument from properties has its origins in Jonathan Lowe (1996; 1998, Chapter 12; 2012). Strictly speaking, Lowe has in mind *kinds* rather than *properties*. Both kinds and properties are taken to be universals and abstract. The difference is that kinds (e.g., appleness) have their instances in objects (an apple), whereas properties (e.g., redness) have their instances in qualities of objects (the particular redness of the apple). The example Plantinga gives— horsiness—is in fact what Lowe would count as a kind. The argument is very connected to the argument from numbers: the universals Lowe has in mind *are* the numbers, and some of the moves made in the argument will end up informing the argument from numbers.

The broad outline of the argument runs something like this: some abstract beings necessarily exist; abstract beings necessarily depend on concrete beings; so, there must be some concrete beings. Universals enter the scene in the first premise. Numbers are supposed to be the necessary ingredients of the necessary truth of mathematics, and Lowe takes numbers *to be* universals, in particular kinds of sets: single-membered sets instantiate the number 1; double-membered sets instantiate 2, etcetera. There are some advantages over the more traditional position that numbers are sets (see Lowe 1998, 220–227; Maddy 1991, Chapter 3). But it wouldn't matter if we took numbers to be sets instead.

Which brings us to the second premise. This is based on three principles: that the only possible abstract beings are sets and universals; that sets must depend on non-sets (as their ur-elements); and that universals must depend on non-universals (as their ur-instantiations). Forget how these principles are supported; you won't be convinced—so long as you think other kinds of abstract beings, or the empty set, or uninstantiated universals are so much as *possible*.

The rest of the story: assume sets or universals could have existed alone; then the sets would depend on the universals (for their ur-elements) and the universals on the sets (for their ur-instantiations); which is an impossible circle of dependence; so sets or universals could not have existed alone. Since sets and universals are the only possible kinds of abstract beings, there could not only be abstract beings. Which is to say that abstract beings depend on concrete beings.

Abstract beings, whether construed as universals or sets, are not possible without concrete beings. Since some abstract beings are necessary beings there must be some concrete being or other. Some concrete being *or other*. The abstract beings might depend on cats; but, if cats never existed, they'd depend on dogs. As it stands, the argument does not point toward any single necessary concrete being.

As it turned out, Lowe was also keen about the ontological argument, and subsequently reworked his argument into what he called "a modal ontological argument," though it is not very much like Plantinga's version. Just reject the option that the necessary abstract beings could depend on contingent concrete beings. The idea that necessary beings could depend on contingent beings is weird:

> to contend that the existence of a necessary being, *N*, is explained in different
> possible worlds by different contingent beings in those worlds threatens to

undermine the very necessity of *N*'s existence. For then it appears to be a mere cosmic accident that every possible world happens to contain something that is, allegedly, able to explain the existence of *N* in that world. (2012, 65–66)

An accident in the cosmoi of possible worlds is less likely than any ordinary cosmic accident. But why would it be a cosmic accident that every world contains a concrete being? I suspect: because nothing is preventing the non-existence of all concrete beings. We can't explain why there had to be contingent beings in terms of contingent beings, and we can't explain it in terms of the necessary beings either—since the necessary beings are supposed to depend on the contingent beings in turn.

There's a similar problem with the original argumentative context: Lowe (1996, 1998, Chapter 12) was trying to explain why there is something rather than nothing—something concrete rather than nothing concrete. Lowe answered that there had to have existed some concrete being or other, even while there might not have been any necessary concrete being. However, we can't explain concrete beings in terms of abstract beings if those abstract beings depend on the concrete beings in turn. That's an impossible circle of explanation and dependence. There being necessary abstract beings might entail there necessarily being concrete beings, which entails there being concrete beings in turn. Entailment isn't explanation, though.

Settle with a necessary concrete being instead. And, it turns out, a necessary intellect. Lowe ends up taking abstract beings to depend on intellectual activity. Just like Plantinga does. Here's the relevant part from Lowe:

> A clue here is provided by the very expression "abstract". An abstract being . . . is one which, by its very nature, is in some sense abstracted from—literally, "drawn out of, or away from"—something else . . . any such being may reasonably be supposed to depend for its existence on that from which it is "abstracted." All the most plausible examples of abstract beings are, interestingly enough, entities that are in a broad sense, objects of reason—such entities as numbers, sets and propositions. . . . But then we have a very good candidate for the sort of being "from" which such entities may be supposed to be somehow "abstracted": namely, a mind of some kind. . . . Putting these thoughts together— (1) that necessary abstract beings, insofar as they are objects of reason, are "mind-dependent", and (2) that they are dependent for their existence on a necessary concrete being—we are led to the conclusion that the being in question must be a rational being with a mind and, indeed, with a mind so powerful that it can comprehend all of mathematics and logic. (Ibid, 69)

Lowe's argument is connected to the argument from numbers. In more ways than one. But the argument need not depend on the existence of numbers, let alone their necessary existence: the necessary existence of any abstract being—any universal or set—would do.

Lowe doesn't say much more than Plantinga does about how abstract beings depend on intellectual activity. Let's see how this can be worked out, moving on to the argument from numbers. I promised that I'd develop the argument so that it shows more about the divine attributes.

Developing the Argument

To remind ourselves, here's the argument from Plantinga:

> It…seems plausible to think of *numbers* as dependent upon or even constituted by intellectual activity; indeed, students always seem to think of them as "ideas" or "concepts", as dependent, somehow, upon our intellectual activity. So if there were no minds, there would be no numbers. . . . But again, there are too many of them for them to arise as a result of human intellectual activity.

Plantinga's argument for the existence of God is reminiscent of Berkeley's. Just as the phenomenal world outstrips our perceptual capacities (where does it go to when we aren't looking?), so too does the mathematical world outstrip out intellectual capacities. In both cases, an infinite intellect is invoked to do the work. Gödel too might have reasoned something like this from our inability to grasp the set theoretic hierarchy to the existence of superior immaterial minds—monads. Since Gödel believed in God, he could presumably have had recourse to the divine mind too. The details about Gödel's views on monads are obscure (see Maddy 1990, 78–79).

Let's assume that numbers do depend on intellectual activity: no intellectual activity, no numbers. Let's also assume that there are too many numbers for any finite intellectual activity to do the work: no infinite intellectual activity, no numbers. If there are numbers then, there must be infinite intellectual activity. Quite some assumptions, and quite some conclusion. Still, it doesn't give us a very religious conclusion: the infinite intellectual activity may be done by something or some things less than God. I want to take things further, and push for a necessary being and a single being.

First, a necessary being. Let's assume that numbers are the ingredients or truth-makers for mathematical truths—they're what mathematical truths are about. In fact, let's assume that they're necessarily the ingredients. It's not just that they happen to be the ingredients. However things had turned out, nothing else could've done the job; for example, rocks couldn't have had much more to do with the fact that $1 + 1 = 2$. Let's also assume that mathematical truths are necessarily true. But if numbers are the essential ingredients of mathematical truths that are necessarily true, then numbers exist necessarily. Given our assumptions, then, numbers necessarily exist.

So we already have some necessary beings on the scene. But from here we can get to necessary intellectual activity easily. Let's assume that numbers not only depend on intellectual activity, but that they necessarily so depend. It's not just that they happen to depend on intellectual activity. However things had turned out, nothing else could do the job; rocks couldn't have had much more to do with numbers. Numbers would depend on something tamer if they could. But if numbers necessarily depend on intellectual activity and necessarily exist, then intellectual activity necessarily exists.

Now we have necessary intellectual activity. And where there's intellectual activity there's an intellect. For all that's been said, though, the intellectual activity need not be the intellectual activity of a necessary intellect. However things had turned out, there had to be

intellectual activity, and so there had to be some intellect or other, but maybe no particular intellect had to exist. If things had turned out one way, then there'd be a certain intellect, *Barry*, but if things had turned out another way, there'd be another intellect, *Mandy*, without *Barry* at all.

We can push further for a necessary intellect. If there were no necessary intellect, the numbers could depend on different intellects: possibly on *Barry*, possibly on *Mandy*. But numbers are necessary beings while the intellects are contingent beings, and so that would mean that necessary beings depend on contingent beings. As we've seen, that's puzzling. Suppose the relevant contingent intellect, say *Barry*, didn't exist. Then, to secure the necessary existence of the numbers, *Mandy* would exist in his place. The necessary existence of numbers would be preventing the non-existence of intellects; the numbers would be forcing, so to speak, some intellect or other into being. So the intellect would depend on the numbers even while the numbers depend on the intellect, which is an impossible circle of dependence.

Now we have a necessary intellect—and an intellect infinite enough to generate the infinity of numbers, though not necessarily all-knowing. It might be natural to think that such a powerful intellect would know of other things. After all, punier intellects do have knowledge of various categories (and come along with other things besides, from will to spatiotemporality). However, the analogy would be pathetic, and not entirely wanted. So far as the argument from numbers goes, I see no way to bridge the gap from knowledge of numbers to knowledge of any other things, let alone omniscience.

Nevertheless, I do think that we can push a little further toward another divine attribute—unity. For all that's been said so far, the intellectual activity could be spread among many intellects. There might be infinitely many intellects each housing its own special number. The move toward a single intellect won't be as smooth as any of the moves so far. The argument will now lose adherents. Again: the more that's said, the more room for error. But, as speculative as the following ruminations are, they strike me as having a little plausibility.

The first thought is that a single intellect that is big enough could do all the work. The view that there is a single intellect containing all the numbers is simpler than the view that there are many intellects. On both views, there are as many numbers, but on the first view there are fewer intellects. Ockham's razor slices away all the other intellects. Once we're already this far gone, however, considerations of simplicity might not have so much pull.

The second thought is that numbers are related to each other. I don't just mean that they come together to form sums and products. How they could come together if they are spread among various intellects would be mysterious enough; but then how they come together at all is mysterious. What I have in mind though is the notion that bigger numbers include or contain littler numbers. Think: babushka doll. The number 3 in some way includes 2 and 1; 2 in some way includes 1. This quite different thought might help us understand how numbers come together in other ways.

The thought is captured by set theoretic constructions of the natural numbers—whether we prefer to say, with Zermelo, that $3 = \{\{\{\emptyset\}\}\}$, $2 = \{\{\emptyset\}\}$, $1 = \{\emptyset\}$, and $0 = \emptyset$, or, with von Neumann, that $3 = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$, $2 = \{\emptyset, \{\emptyset\}\}$, $1 = \{\emptyset\}$, and $0 = \emptyset$. I don't know how seriously to take such constructions, and I don't know how seriously someone who takes numbers to be ideas or concepts should. But being able to do something like this is an advantage of a theory. The thought is also captured by Leibniz's definitions: 2 is 1 and 1; 3 is 2 and 1; 4 is 3 and

1; etcetera. When I learned how to count, I was given little colored blocks: the first one for 1, and then another one for 2. The first one remained a part of the scene.

If the intellect generates or contains the big number, and the big number includes the little number, then the intellect must contain the little number too. It's not just that an intellect generating a bigger number can do all the work of many intellects each generating littler numbers; it's that the intellect generating a bigger number *will* do all that work. If there are littler intellects, they're redundant, generating many identical or at least indistinguishable number-ish ideas. We'd be stuck with way too much intellectual activity. Now if there were a biggest number, then we could easily move to an intellect that generates all the other, littler numbers. (The little-held view of ultrafinitism would seem to entail their being a biggest number, though it's probably best not to rely on that view here—or anywhere.) However, as it is, if we posit any intellect that does not generate all the numbers, there will always be too much intellectual activity. Ockham's razor cuts again.

Another point in the same direction: Plantinga actually calls the argument from numbers "The Argument from (Natural) Numbers." But why focus on natural numbers? Maybe we do better by focusing on irrational numbers. After all, it would take a much more impressive intellect to house non-repeating decimals that never end. And it would have to contain all the decimals. We can think about pi: but our incomplete knowledge isn't enough to encompass its full and determinate nature. Maybe all we need is 22/7, though. Or better yet, π. (And maybe Euler thought pi could prove that God exists in another way.) Similarly, we can think of a big number without thinking, at least at all explicitly, about littler numbers. But then our thinking hardly constitutes the very being of these numbers.

Leibniz anticipates some of these moves, but in no way drawing from his definitions of numbers. Leibniz argues not from the existence of numbers, but from the existence of necessary truths to the existence of God—including necessary truths beyond the truths of arithmetic and mathematics.

Leibniz construes necessary truths to be dependent beings, only abstracted in the intellect. This is quite like the way Lowe ultimately construes numbers. And just as the truths are necessarily true, there had to exist some intellect. But the truths can't depend on just some intellect or other, without any being necessary. Leibniz insists that necessary beings can't depend on contingent intellects. Again, like Lowe. But Leibniz doesn't explain why.

The argument from numbers that Plantinga attributes to Adams via Kenny is in fact Leibniz's argument, as developed in Adams's book *Leibniz* (see Adams (1994, Chapter 7)). At this point in the argument, Adams finds recourse to the Principle of Sufficient Reason: "it is hard to see what the sufficient reason would be that would determine *which* of those individually contingent beings would exist, if there is not one of them that necessarily exists with the power to decide which contingent beings shall exist" (1994, 183). So they must depend on a *necessary* intellect. And, since the necessary truths are logically related, that must be a single intellect.

That last part about the relations between necessary truths is especially complicated, and not especially related to the argument from numbers. The argument we've been developing doesn't depend on these very tricky thoughts or on the PSR. While Leibniz was committed to that principle, we might not be. The argument we've been developing is easier than Leibniz's. Easier. But that doesn't mean better, all things considered.

So we've pushed toward an infinite, necessary, and unified intellect. Not yet God, but not bad company either. After all, more famous arguments for the existence of God show no more than this or that divine attribute, and practically a lot less. Compare the cosmological argument from contingency. If it worked, it would show that there is a necessary being on which contingent beings depend. That points to quite robust power, though not to anything at all intellectual. The argument from numbers would show that there is a necessary being on which other necessary beings depend. It points toward an infinite intellect, though not to anything at all powerful.

Now that I've extended the argument, answering a few potential objections along the way, let's turn to the main advantages of the view, and what I take to be the main problems.

Versus Psychologism

Adams, following Leibniz, sees the argument for what we might call divine psychologism in its advantages over the alternatives of run-of-the-mill psychologism and Platonism. As I've noted, the Adams-Leibniz argument differs from Plantinga's in focusing on the necessary truths of logic and mathematics, rather than the existence of numbers. The points apply equally to both. Plantinga's main focus is against human psychologism, which construes numbers as human ideas. Adams calls such a view the "anthropological" theory (1994, 178).

Divine psychologism certainly has various advantages over human psychologism. Human psychologism just won't do for reasons owing ultimately to Frege (1960). If numbers are human ideas, then there'll be too few of them since each of us is finite and there aren't enough of us. If numbers are human ideas, then there won't be any number greater than the greatest number we've thought up. Here's another problem at least close to one put forward by Leibniz about necessary truths generally: if numbers are human ideas, then numbers—and mathematics which has numbers as its essential ingredients—would be just as contingent as we are. But there are enough numbers, and numbers greater than any we've had, and they're not contingent. These points are at work in the argument for divine psychologism developed earlier. Avoid them with an infinite, necessary intellect.

But there are two problems raised by Frege I'm not so sure of: I'm not so sure how serious the problems are, or whether divine psychologism avoids them. The first is about privacy: if numbers are ideas, and ideas are private to thinkers, then we will each have our own numbers: "We should have to speak of my two and your two, of one two and all twos" (1960, Section 27; if only Dr. Seuss wrote the whole book).

If the number 2 is a divine idea, is it any less private than if it is a human idea? Maybe. If 2 is a human idea, then presumably it's as much my idea as yours, so that we have to speak of my 2 and your 2; you're not especially privileged such that we're all talking about *your* idea. However, if it's a divine idea, then we could all be talking about the same idea, God's idea. Divine ideas would have to be relevantly different from our own. They're at least different in that they can secure the infinity and necessity of numbers. But maybe the big difference is that our ideas are confined to our respective minds, so that the best we can do is to sincerely tell each other about our ideas, whereas God has the power to somehow give all of us more direct psychological access to his own ideas. (This might connect with a point below about how divine psychologism helps secure our cognitive contact with numbers.)

The second problem from Frege is more serious. This is a kind of contingency problem. It's not about the numbers being contingent, but about the mathematical truths that result from them being contingent: "As new generations of children grew up, new generations of twos would continually be being born, and in the course of millennia these might evolve, for all we could tell, to such a pitch that two of them would make five" (Ibid; see also Frege 1960, xvii–xix). If numbers are human ideas, then mathematics is as contingent as we are and as our thinking is. Now $2 + 2$ happens to equal 4, but if we come to think differently, $2 + 2$ will equal 5—and if we had thought differently, it could have equaled 5. This is absurd.

We might think that divine psychologism can answer the problem as easily as it can answer the earlier problem about the contingency of numbers themselves: the necessary nature of the numbers as well as their necessary existence could be secured by the necessity of the divine intellect. And we might be right. But it's not clear that either problem is solved. It could be that God would always have had the ideas of 2 and 4, and it could be that God would always have had the idea that $2 + 2 = 4$, no matter what. But perhaps it could be that God would not necessarily have the ideas. Why should he have them? If God could have had quite different ideas (or none at all), it would be a cosmic coincidence for him to choose just those ideas, no matter what.

Let's focus on the main subject of Leibniz's argument first: the necessary truths of mathematics. Whenever we try to ground some domain in the divine, there's a Euthyphro-style dilemma lurking. A comparison here between morality and mathematics might be illuminating. The traditional Euthyphro dilemma: Does God command it because it's obligatory? Or is it obligatory because he commands it? If the former, then there's some morality independent of God's say-so. If the latter, then God could with as much reason command murder as he could forbid it. Does God think that $2 + 2 = 4$ because $2 + 2 = 4$? Or does $2 + 2 = 4$ because he thinks it? If the former, then the truth is independent of God's say-so. If the latter, then God could with as much reason have decided that $2 + 2 = 5$. Descartes was happy enough with such prospects. A bonus: God could have solved Christian theologians a good deal of puzzlement by simply thinking that $1 = 3$.

Leibniz rejected the divine command theory for a couple of reasons, including this: "it seems that all acts of will presuppose a reason for willing and that this reason is naturally prior to the act of will" (1991, 2). What God wills must be guided by reason, and so whatever God wills about morality must be guided by reason, and it's in that reason, rather than God's will, where morality ultimately lies. Similarly, should Leibniz not have seen that what God wills about mathematics must be guided by reasons, and mathematics is ultimately to be found in those reasons?

Return to the main subject of Plantinga's argument: not the necessary truths of mathematics, but the numbers. Draw the parallel conclusion: Does God have the idea of 2 because it exists? Or does it exist because he has the idea? If the former, then the number is independent of God's intellect. If the latter, then God could with as much reason never have had the idea of 2, so that it never existed. Assuming that the numbers necessarily exist, that won't do.

In a way, though, it's less immediately problematic than the implication that God could have decided that $2 + 2 = 5$. If the essential ingredients for $2 + 2 = 4$ did not exist, then it wouldn't have been the case that $2 + 2 = 4$. But it wouldn't have been the case that $2 + 2 = 5$ either, because the essential ingredients for that wouldn't have existed either. The view that it could have been that it is not true that $2 + 2 = 4$ is weird, but the view that it could have been that

2 + 2 = 5 is even weirder. So the problem might not be as pressing for Plantinga's argument—which is not to say that it won't be pressing.

There might be a way out of the problem altogether. The Euthyphro-style dilemmas trade on the subject (whether morality or mathematics) depending on God's "commands" or "say-so" or "will," as I've variously put it—something that's supposed to be to some extent contingent. And that's a natural way to construe ideas. They're things subjects come up with and are free to play around with. God can then come up with and play around with his ideas about numbers as much as we can with our idea of a friendly mouse who wears red pants, yellow shoes and white gloves. That might be the wrong way to think about God's ideas.

Adams (1979) and Alston (2002) try to avoid the Euthyphro dilemma for morality by invoking a *loving* will: a loving will won't command murder. We might avoid the Euthyphro dilemma for mathematics by invoking what we might call a *rational* will: a rational will won't lack the idea of 2 and won't think that 2 + 2 = 5 either. If the will is as necessary as it is necessarily loving and rational, then morality and mathematics couldn't have gone awry. God's ideas about numbers are then necessitated by his rational nature, and are not so much a matter of will (as that term is usually understood) at all.

But I wonder: Why would God's rational nature ensure that 2 + 2 = 4? Unless there's something intrinsically rational about 2 + 2 = 4, unless there's something about the numbers that God's rationality is tracking and that isn't up to anyone, God could have dreamt up something else. But, if God's rational nature is tracking something, then that is what mathematics is about. *That* is where the numbers live. It might be a luminous Platonic realm. It might be next to nothing at all. There might even be something divine about it.

In a similar connection, Alston answers that God's commands are tracking his loving nature. This good nature is in a way prior to the divine commands. There is then value independent of the divine commands. Yet, a Euthyphro-style dilemma need not arise again here. For the fundamental ingredients of morality and value need not be anything independent of God. God can be taken to play the same role a Platonic form of goodness is supposed to. We might try a similar solution to the Euthyphro-style dilemma about mathematics: God's rational will could be tracking something about God. Then the metaphysics for mathematics need not be anything independent of God.

Unfortunately, I don't know how to fill this picture out any further; I don't know what exactly it is about God's nature that's doing the work in our case. The hypothesis that there is a relevant something-we-know-not-what might indeed be the start of an answer to the Euthyphro-style dilemma about mathematics. But don't then be too hard on Platonist opponents of divine psychologism should they appeal to their own we-know-not-whats in defense of Platonism.

My main worry is that leaving things at a mystery comes at a particular cost in the context of the argument from numbers—again a problem about how much of an audience the argument might attract. Even if the maneuver has the benefit of answering the Euthyphro-style dilemma, it costs a premise of the argument from numbers in plausibility. The premise is that numbers depend on intellectual activity, that they are ideas. But, in order to avoid the Euthyphro dilemma, it turns out that the intellectual activity and the ideas are very far from what was originally imagined, and indeed that the numbers are not so much ideas as something quite unknown that the ideas track.

What attracts Plantinga's students to psychologism in the first place? I don't really know. Here's a little speculation: numbers are elusive things. They strike some of us at first as not quite real, but not quite unreal either—"real" and "unreal" in the metaphysical, not the mathematical, sense! They strike some of us as neither the inhabitants of the material realm, nor the inhabitants of a Platonic realm. But the mind lies somewhere between these realms: our ideas and thoughts aren't as heavy and thick as material beings, but aren't as light and thin and distant as Platonic beings. The intellect promises to be the perfect environment for numbers.

As we've seen, though, not just any intellect will do: we need something like a divine intellect. However, once we've reached divine psychologism, the original appeal of psychologism is risked. The divine intellect might not be much more down to earth than the Platonic realm. If the appeal for psychologism was to avoid the mysteries of Platonism, then the divine intellect, with its own mysteries, might not do much better. Is there enough resemblance between God's ideas and our own ideas to secure whatever was originally attractive about identifying numbers with ideas? "For my thoughts are not your thoughts" (Is. 55: 8).

Versus Platonism

So much for the advantages of divine psychologism over human psychologism. Adams says less of Leibniz's rejection of Platonism. But it has to do with his conception of the objects of logic and mathematics not being the kinds of beings the Platonist construes them as—beings that could subsist independently in a Platonic heaven. They are instead for Leibniz "modes": "the basic idea is that impossibilities, truths, natures and essences, and other objects of logic are abstract objects in the original sense; that is they can be conceived only by abstractions from a richer, more complete being or 'subject'" (cited in Adams 1994, 180). Recall Lowe. Logical and mathematical beings are dependent beings—dependent on the divine mind.

Plantinga makes the point in a more recent statement of the argument from numbers:

Now there are two quite different but widely shared intuitions about numbers and sets. First, we think of numbers and sets as abstract objects. . . . On the other hand, there is another equally widely shared intuition about these things: most people who have thought about the question think it incredible that these objects should just exist, just *be* there, whether or not they are thought of by anyone. Platonism with respect to these objects is the position that they do exist in that way. . . . It is therefore extremely tempting to think of abstract objects as ontologically dependent upon mental or intellectual activity in such a way that either they just are thoughts, or else at any rate couldn't exist if not thought of. (2011, 288)

Again, numbers can't have an independent being, and can't have a being independent of thought in particular. Once we've ruled out Platonism, the rest of the story is familiar. We rule out human psychologism in favor of divine psychologism: "if it is human thinkers that are at issue then there are far too many abstract objects. . . . On the other hand, if abstract objects were divine thoughts, there would be no problem here. So perhaps the most natural way to think of abstract objects, including numbers, is as divine thoughts" (ibid). All I can say is that I don't have the intuition or face the temptation Plantinga mentions. The term *abstract* did originally have connotations of something abstracted in thought. But now it means something else: now it means non-

spatiotemporal or non-causal. So far as I can tell, neither Leibniz nor Adams nor Plantinga nor Lowe provide a real reason here for thinking that such beings must depend on minds.

However, Leibniz, Adams, and Plantinga have another reason—not so metaphysical as epistemological. The most famous problem for Platonism is about our knowledge of mathematics (see Benaceraff 1965). The problem is really tricky when we have a Platonic heaven of impotent numbers: how do we make cognitive contact with such things? It might be a little less tricky if the numbers are in actual heaven. God could be powerful enough to forge the connection that allows us to talk about his ideas. Adams cites Leibniz approvingly: "just as God is the original source of all things, so also is all fundamental knowledge to be derived from God's knowledge, and in his light we see light" (1994, 187).

Plantinga similarly sees the solution to the problem of cognitive contact in theism. According to divine psychologism, numbers:

> would stand to God in the relation in which a thought stands to a thinker. This is presumably a productive relation: the thinker produces his thoughts. It is therefore also a causal relation. If so, then numbers . . . stand in causal relation to us. For we too stand in a causal relation to God; but then anything else that stands in a causal relation to God stands in a causal relation to us. (2011, 291)

The earlier worry about the contingency of the divine thoughts about numbers emerges again with the idea that God causally produces thoughts. That looks like contingency. But there is, again, another worry about whether we make any progress with divine psychologism. If it's hard to see how we make contact with transcendent numbers, then will there not be the same problem about how we make contact with a transcendent God or his thoughts?

However, I do not think the problems are the same. The problem about abstract beings, as usually conceived, is that they are not causal. But God, as usually conceived, is causal par excellence: God is supposed to be able to do anything. There may be other relevant differences. Abstract beings are supposed to be perfectly spaceless and timeless. God's relation to space and time is a matter of philosophical controversy; many contend that God exists in time, and some even contend that he exists in space. If absolute transcendence is what makes for the impotence of abstract beings, then God might have an advantage so long as he is not so absolutely transcendent.

The usual objections against substance dualism—that it's hard to see how an immaterial mind could interact with a material body—are usually no more than mere assertions. Exercise: pull a philosophy of mind book off the shelf and check whether the rejection of interaction on substance dualism is anything more than a mere expression of puzzlement. The critics do well to stick to mere puzzlement; whenever an argument is given it is extremely weak (see Lycan 2009; Plantinga 2007b, 124–33; Hoffman & Rosenkrantz 1994, Chapter 5; Foster 1991, Chapter 6). As much was to be predicted: there is no plausible theory of causation that rules out causal interaction on substance dualism—and not the least because there is no plausible theory of causation.

Furthermore, some of the more serious problems for such interaction—problems about our minds puncturing the causal closure of the physical realm—do not begin to apply to a divine mind who would have brought about and would sustain the entire physical realm. While finite

immaterial minds might be constrained by the tight network of physical causes and laws, an infinite immaterial mind would not be so constrained.

But is there really a problem with Platonism to begin with? Platonists have put forward answers, answers that do not invoke God (see Balaguer 1998, Chapters 1 & 2). Nominalists aren't convinced. Perhaps they could find resources in divine psychologism. Then again, the motivations for nominalism might tell against divine psychologism too. These motivations include a more generally naturalistic or materialistic philosophical conscience. Naturalism and materialism rule out God just as much as they do Platonic numbers. Thus James Brown's friends tease him for being both a Platonist and an atheist:

> along the lines that being a Platonist is really no different from believing in God. I'm kidded about being soft on superstition, a closet religionist, and so on. While I enjoy the kidding I've actually never seen the slightest connection between religion and mathematical Platonism. . . . Polkinghorne favorably cites a number of prominent mathematicians who are also Platonists (Gödel, Hardy, Connes) in support of the notion that mathematics is transcendent. God, of course, is transcendent, too, so, Polkinghorne seems to suggest, there is a kind of mutual support—Platonists should believe in God. (2012, 161)

Polkinghorne's idea, apparently, is that once you've let in one kind of transcendent being, you might as well let in another. And Brown's friends agree on that point. "But," answers Brown, "aside from transcendence, there is no real connection between belief in God and belief in a Platonic realm" (ibid). Divine psychologism would provide another connection in answering the problem of cognitive contact. But Brown would not be moved. He doesn't think there is any epistemological problem for Platonism to begin with: he rejects the very premise of the problem, that knowledge requires causal contact.

Others might find the problem of cognitive contact more pressing for Platonism, and make recourse to divine psychologism. However, as we have seen, not many naturalists can consistently do so, and it's naturalists who find the problem most pressing. Again, the audience for Plantinga's argument will dwindle. But here it need not be entirely lost: if they are naturalists because of the problem of cognitive contact with the transcendent, they might be convinced that there is less of a problem about contact with a divine realm than about contact with a Platonic realm; and Platonists who see some problem with cognitive contact might come to see less of a problem on theism. Theism could become an option. Insofar as naturalists are naturalists because they have trouble understanding causal contact between the natural and the transcendent, they should not be naturalists. That is no real support for naturalism. But naturalists tend to have other reasons besides.

Conclusion

Plantinga (and Adams and others) see divine psychologism as having advantages over both human psychologism and Platonism. Human psychologism can be ruled out completely as a contender. However, what rules it out might rule out divine psychologism too. Platonism is still a contender. There is a similar worry that the main problem with Platonism will also be a problem

with divine psychologism; however, it will, at the least, be less of a problem. Of course, psychologism and Platonism are not the only alternatives. There are views that have not been addressed here at all. For example, there is the fictionalist view that numbers do not exist at all. Within the scope of an essay, I've just been able to touch on the main points raised by Plantinga.

Finally, so far as I can see, none of this touches other arguments about numbers for the existence of God—arguments not from the existence of numbers, but from their applicability to science or from our ability to do really hard math (see Plantinga 2011, 284–291; Steiner 1998). Even if the existence of numbers has nothing to do with God, it still might be that "[t]he miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve" (Wigner 1960, 14). Well, the main idea here is that on theism it is *not* so hard to understand. The question of God and mathematics in general—quite unlike the question of God and science or the question of God and morality— has not been very much explored. It all needs some more "loving development."[1]

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