Outdoor Mathematics

Applications and Ideas for
High School Mathematics
In or about the Outdoors

Presentation at
North Carolina Council of Teachers of Mathematics Annual Meeting

October 11, 2007

by
Mathematics Education Students and Faculty
Wake Forest University

- **Angles of Elevation and Depression by Austin James**
- **Regression Relay by Ashley Lumpkin**
- **Graphing Goes Live by Lauren Brooks**
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Angles of Elevation and Depression

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Audience: Geometry or Trigonometry Students

Objectives:

- Students will be able to apply sine, cosine and tangent ratios to find angles of elevation and depression.
- Students will be able to measure lengths and use measurements to determine angle measures.

Standards:

North Carolina Standard Course of Study:

- Geometry – Goal 1.01 – Use trigonometric ratios to model and solve problems involving right triangles.
- Advanced Functions and Modeling – Goal 2.04 – Use trigonometric (sine, cosine) functions to model and solve problems.
- Pre-Calculus – Goal 2.02 – Use trigonometric and inverse trigonometric functions to model and solve problems.

National Council for Teachers of Mathematics:

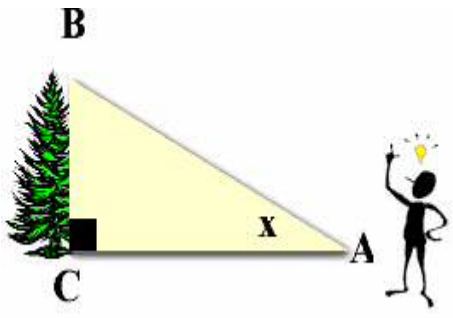
- Standard 2: Understand functions.
- Standard 4: Apply proper formulas to find measures.

Process Standards:

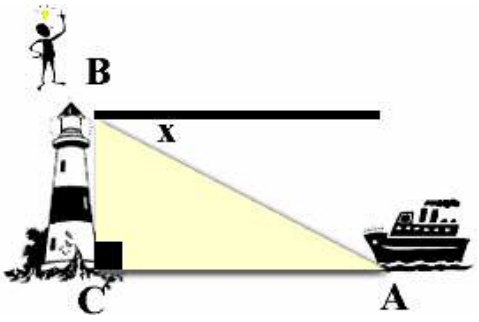
- Problem Solving, Connections, Representations

Explanation:

Trigonometric ratios have many practical real-world examples. Angles of elevation and depression are formed by the horizontal lines that a person's lines of sight to an object form. If a person is looking up, the angle is an elevation angle. If a person is looking down, the angle is a depression angle.



x = angle of elevation from ground to top of tree



x = angle of depression from lighthouse to boat

(Pictures taken from http://regentsprep.org/regents/Math/rtritrig/LtrigW.htm)

In this activity, students will use trigonometric ratios to determine the angle of elevation to the sun and the height of an unknown object. Student groups will use the height of a person and the length of his or her shadow to find the angle of elevation to the sun. Then, the students will use that angle of elevation and length of an unknown object's shadow to find the height of that object. *This activity gives the students two different applications of trig ratios.* In my example, I will use a football goalpost as the unknown object.

Preparation:

- Students should be comfortable with trig ratios and inverse trig ratios.
- Make sure you do this activity on a sunny day. Without shadows, the activity will not work.
- Students will need tape measurers to determine necessary lengths. If the school does not have tape measurers, ask the students to bring them from home (potentially for bonus points).
- Depending on the class size, the teacher may need to have more than one unknown object for the students to find the heights of. A sports field should have lots of options – goalpost, goal, flag pole, etc.
- It is not necessary that students have any formal definition of angle of elevation and depression. This activity could be used as an introduction to the concept (discovery) or as a supplement to previous instruction.

Activity: (students should be in groups of up to four people)

1. Measure the height of one person in the group.
2. Measure that person's shadow.
3. Using what you know about trig ratios, determine the angle of elevation from the ground to the sun.
   (Students should sketch a picture of the situation to help make sense of the computation.)
4. Measure the shadow of the object.
5. Using the angle of elevation and the measure of the shadow, use what you know about trig ratios to determine the height of the object.
   (Students should sketch a picture of the situation to help make sense of the computation.)
6. Fill in all of the information in your picture (you should now have two right triangles with 2 sides and an angle measure).
   *** Depending on what measuring devices you have, tell your students what units to measure in ***

Assessment Possibilities:

- Ask the following question and have the students journal:
  Can the sine or cosine of an angle ever be greater than 1? If so, when? If not, why?
- Have the students draw and describe their own application of angles of elevation and depression, complete with a solution.
- Give the students a few word problems with scenarios similar to the outside experience they did in class. The students should draw a picture and solve.

What can I learn from my SHADOW ?

Name: _____ **Date:** _____

Group Members: _____

Objective: Students will apply trigonometric ratios and other things they know about right triangles to determine the height of an object outdoors.

Trig Ratios:

We have used right triangles to determine some important relationships that you have listed above. Today, you and your group members will use these ratios to determine the height of an object outside. Follow the following steps:

1. Pick one person in the group and measure height: _____

   Name of person you are measuring: _____

2. Measure the length of that person's shadow: _____

3. Using the appropriate trigonometric ratio, find the angle of elevation (sketch picture):

4. Find the length of the shadow of the object your group has chosen: _____

5. Using the angle of elevation and the shadow length, find the height of the object:

   _____

6. Sketch a picture of the object, its shadow, and the angle of elevation.

What can I learn from my SHADOW?

Name: John Doe **Date:** 12-12-2012

Group Members: Mark, Roberto, Tasha

Objective: Students will apply trigonometric ratios and other things they know about right triangles to determine the height of an object outdoors.

Trig Ratios: $\sin = \frac{opp}{hyp}$ $\cos = \frac{adj}{hyp}$ $\tan = \frac{opp}{adj}$

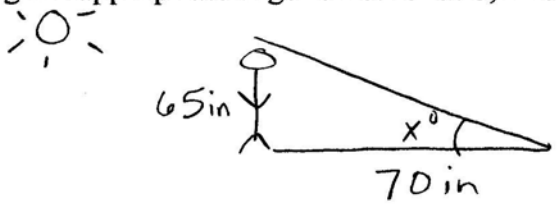
We have used right triangles to determine some important relationships that you have listed above. Today, you and your group members will use these ratios to determine the height of an object outside. Follow the following steps:

1. Pick one person in the group and measure height: 65 in

   Name of person you are measuring: Tasha

2. Measure the length of that person's shadow: 72 in

3. Using the appropriate trigonometric ratio, find the angle of elevation (sketch picture):



$\tan x^\circ = \frac{65}{70}$

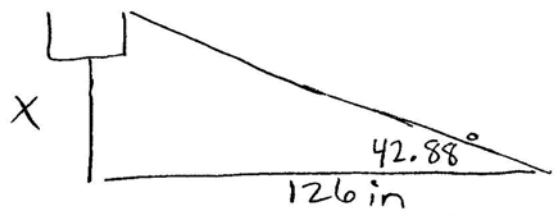
$x = 42.88^\circ$

4. Find the length of the shadow of the object your group has chosen: 126 in

5. Using the angle of elevation and the shadow length, find the height of the object:

   $\tan 42.88 = \frac{x}{126}$ $x = 117.004$ in

6. Sketch a picture of the object, its shadow, and the angle of elevation.



Regression Relay

Ashley Lumpkin
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NCTM STANDARDS: Number and Operations, Algebra, Measurement, Data Analysis & Probability, Connections

NC STANDARD COURSE OF STUDY: Algebra 2, objective 2.04

GOALS: Students will be able to use graphing calculators to fit regression models to a given set of data. They will be able to interpret slope and y-intercept, and use the model to make long term predictions.

INTRODUCTION:

The relay race is the only true team event in track and field, and is traditionally used to close track and field events. They can very in length; sprint relays include the 4x100m, 4x200m, and long distance relays include the 4x800m and 4x1200m. The only relays at the Olympics and world championships are the 4x400m and 4x100m. Each race is split into four sections called legs, with each of four runners running one leg of the race, and then passing the baton to the next runner.

We modify the traditional relay race to help students collect and interpret linear data.

MATERIALS: stopwatches, a "baton" (the cardboard tube from a paper towel roll is perfect, but anything light will do) activity sheets

LESSON DEVLOPMENT / GUIDED PRACTICE

IN THE CLASSROOM:

- Introduce the activity
- Determine the length to be run in each trial.
- Determine which students will participate in the activity; you need 10 runners and 3 time keepers,
- Determine how discrepancies in time will be handled:
    - Will there be an "official timekeeper" to be defaulted to?
    - Will the average of the different times be used?

OUTDOORS

- Measure and mark the length to be traveled during each run.

- Students will fill out the chart at the top of the activity sheet as each trial is run:
    - At the timers command, one student will run to the set mark, and then run back to the start line.

    - The time will stop when he crosses the start line; this time will be recorded in the running column, in the row marked for 1 runner

    - Repeat with 2 students, 3, 4, 5, and 10 until the chart is complete.

    NOTE: Because the first runner also runs in each of the other "heats" (as does the second in the final 4, and so on), it may be best to encourage students not to sprint full out (as they'll be compelled to do), but to jog at a consistent pace, especially if they will running a considerable distance.

BACK IN THE CLASSROOM:

- Students should work together to complete through # 8 on their activity sheets
    1. Entering data into calculator
    2. Finding the regression line
    3. What is the equation of the line?
    4. What is the slope?
    5. What is the y-intercept?
    6. What does each of those values mean in terms of the data?
    7. Sketch the graph of the plotted points and regression line.
    8. Predict how long it would take for the entire class to run the relay

ACTIVITY EXTENSION:

It may be interesting to see how slope and intercept are affected for types of travel besides running (e.g. walking or skipping). If you have more time to devote to the activity, have the students walk, skip, and some other form of movement. They can work in groups with one of the four sets of data, then as a class compare and contrast the results. (A modified activity sheet is provided if you choose to do the activity in this way)

INDEPENDENT PRACTICE

The USA Track and Field (USATF) website gives the rankings of various athletes in various track and field events. Using the information from the site, have students create a 4x400 m relay "dream team".

- The records portion of USATF can be found at http://www.usatf.org/groups/TrackAndField/statistics.asp
- Students can select male/female athletes, from either indoor/outdoor events
- Scroll down the 400m times, then select 4 athletes
    - The same athlete cannot be used twice (that is, your team cannot be comprised of the same athlete's times in different races)
    - The times do not have to occurred in the same race
- Using these athletes and their times, they should make a scatter plot, with the number of runners as x-values, and the cumulative time of the runners as y-values
    - Ex: from the site http://www.usatf.org/statistics/topMarks/2006/outdoorTF/men.asp if my first two athletes are Jeremy Wariner and LaShawn Merritt, then I should have the points (1, 43.62) followed by (2, 87.76) where 87.76 is the combined time of 43.62 (Warriner's time) and 44.14 (Merritt's time)
- The students should use this data to complete the remainder of their activity sheets individually

ASSESSMENT: Assessment is the completed worksheet.

Regression Relay
Activity Sheet

Number of Students	Time
1	
2	
3	
4	
5	
10	

1. **Enter the data into your calculator, and make a scatter plot**

    - To do this, hit STAT and select 1: Edit
    - If you have anything in L1 and L2, arrow up to the name of the list, hit CLEAR and ENTER.
    - Enter in your values one at a time by pressing ENTER after each number (The number of students should be in L_1 and the time should be in L_2).
    - Turn your stat plot on. To do this, hit 2nd Y =, select Plot1 BE SURE THAT
        - that ON is highlighted
        - the scatter plot is highlighted,
        - X list is set to L1, and Y list is set to L2.
    - Now press ZOOM and select 9: Zoom Stat. The calculator automatically chooses an appropriate window so you can see all of the data points

2. **Find the regression line**

    - To find the regression equation, first hit STAT, arrow right to CALC and select option 4: LinReg (ax + b). Now press 2nd 1, 2nd 2, to specify where you entered your data. (Don't forget the commas). To store the function into Y1 press VARS, arrow over to Y-VARS, choose FUNCTION and then select Y1.
    - Your screen should now read LinReg (ax + b) L1, L2, Y1. If it does press ENTER to see the equation of the regression line.

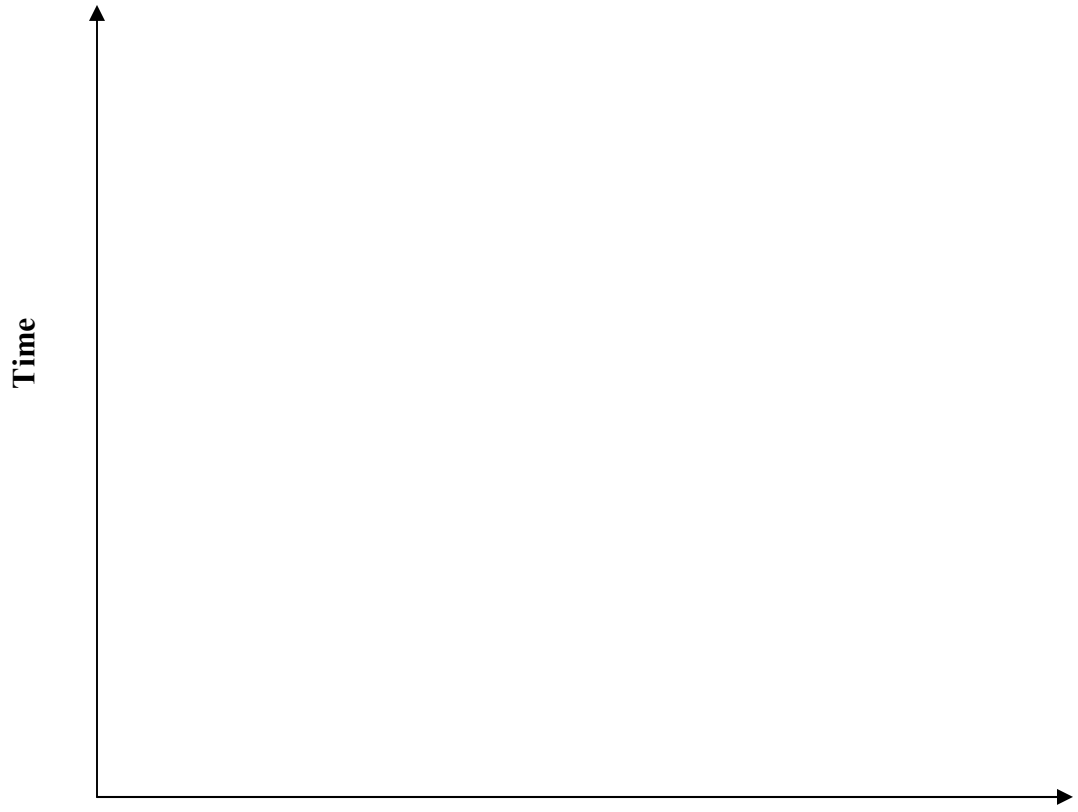
3. **What is the equation of the line?**

4. **What is the slope of the line?**

5. **What is the y-intercept of the line?**

6. **What do the slope and y-intercept mean *in terms of your data*?**

7. **Sketch a graph, including the data points and the regression line. (Don't forget to draw a scale on both axes)**



8. **From the graph, how long would it take for the entire class to run our relay?**

ON YOUR OWN: "Making the Dream Team"

The USA Track and Field website gives the rankings of athletes in various track and field events. (http://www.usatf.org/groups/TrackAndField/statistics.asp) Use the information from the site to make a 4x400 m relay Dream Team.

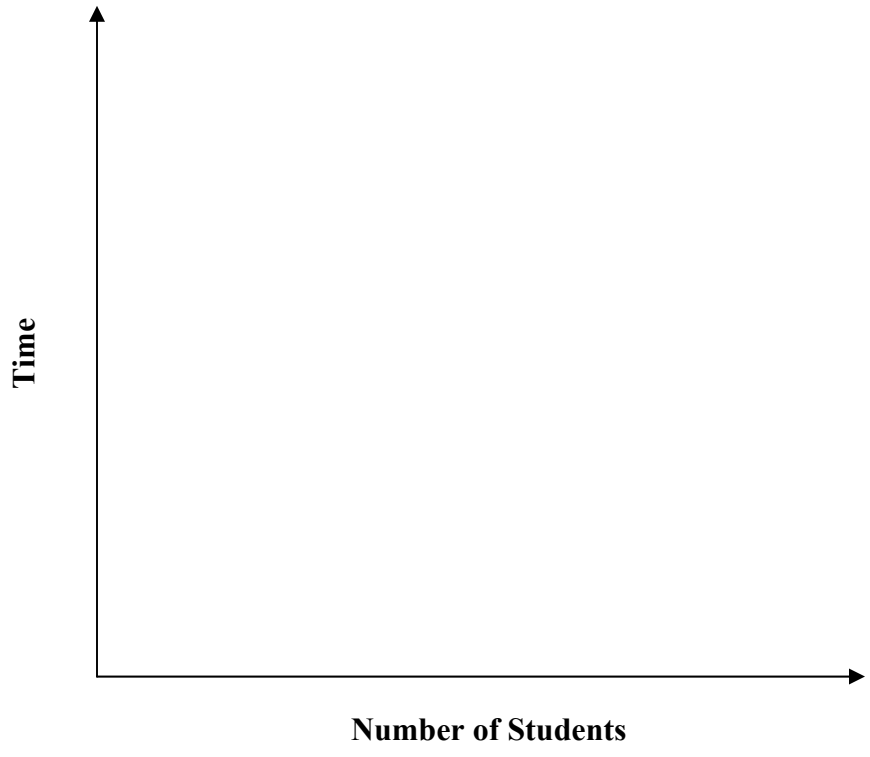
9. Complete the following table

Athlete		Time	Cumulative Time

10. Make a scatter plot of the data: (L1 should be 1,2,3,4; L2 your cumulative time column)

11. Find the regression line

12. Sketch a graph, including the data points and the regression line. (Don't forget to draw a scale on both axes)



13. From the graph, how long would it take for the entire class to run our relay?

14. Sketch the graph.

15. What is the slope of your line?

16. What is the y-intercept of your line?

17. From your line, how long will it take for 25 Olympians to each run 400 meters?

Regression Relay
Activity Sheet

	Time							
Number of Students		***Running***		***Walking***		***Skipping***		***Other*** _____
1								
2								
3								
4								
5								
10								

In the chart, highlight the column of data you have been assigned to work with

1. Enter the data into your calculator

- To do this, hit STAT and select 1: Edit
- If you have anything in L1 and L2, arrow up to the name of the list, hit CLEAR and ENTER.
- Enter in your values one at a time by pressing ENTER after each number (The number of students should be in L_1 and the time should be in L_2).
- Turn your stat plot on. To do this, hit 2nd Y =, select Plot1 BE SURE THAT
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2. Find the regression line

- To find the regression equation, first hit STAT, arrow right to CALC and select option 4: LinReg (ax + b). Now press 2nd 1, 2nd 2, to specify where you entered your data. (Don't forget the commas). To store the function into Y1 press VARS, arrow over to Y-VARS, choose FUNCTION and then select Y1.
- Your screen should now read LinReg (ax + b) L1, L2, Y1. If it does press ENTER to see the equation of the regression line.

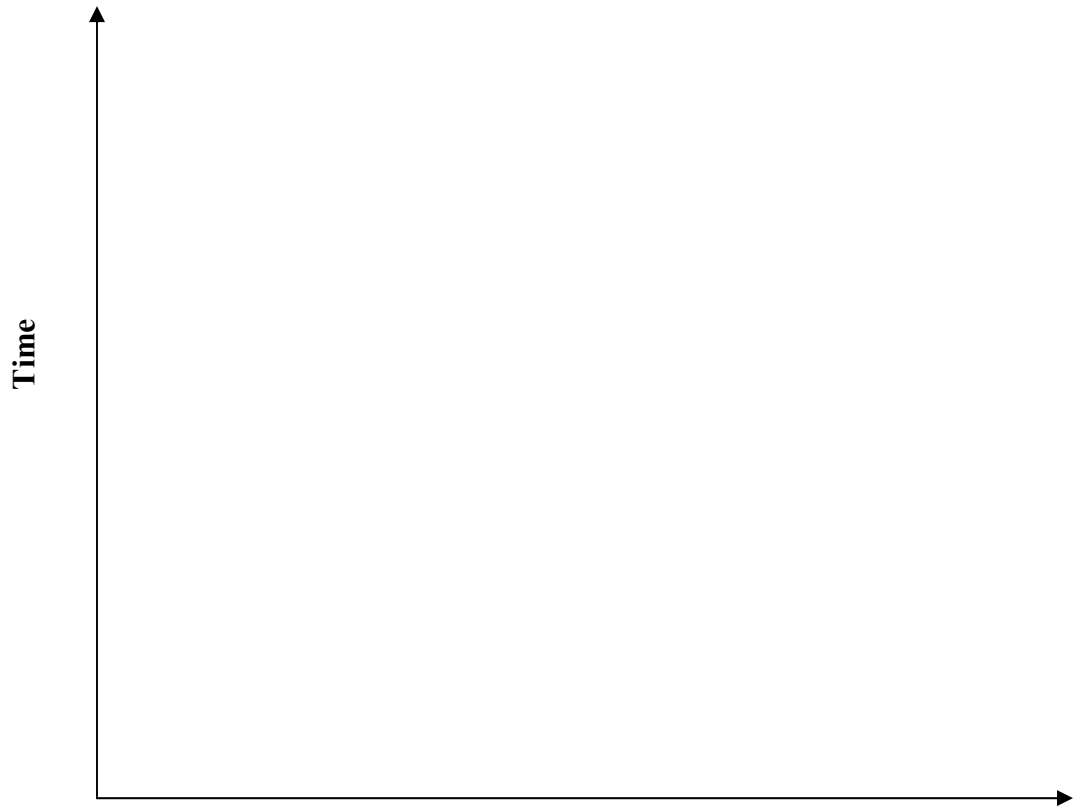
3. What is the equation of your line?

4. What is the slope of your line?

5. What is the y-intercept of your line?

6. What do the slope and y-intercept mean *in terms of your data*?

7. Sketch a graph, including the data points and the regression line. (Don't forget to draw a scale on both axes)



8. From the graph, how long would it take for the entire class to run (walk, skip, or _____) our relay?

9. (As a class) Complete the following table:

Style of Movement	Regression Line	Slope	Y - Intercept
Running			
Skipping			
Walking			
_____			

10. How are the graphs similar? How are they different?

ON YOUR OWN: "Making the Dream Team"

The USA Track and Field website gives the rankings of athletes in various track and field events. (http://www.usatf.org/groups/TrackAndField/statistics.asp) Use the information from the site to make a 4x400 m relay Dream Team.

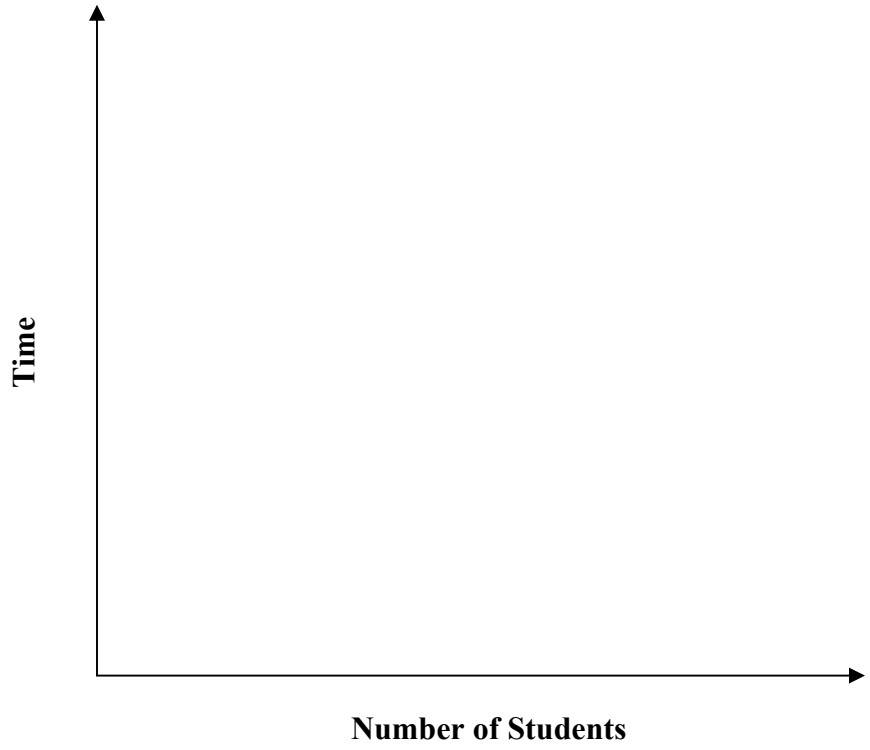
18. Complete the following table

Athlete		Time	Cumulative Time

19. Make a scatter plot of the data: (L1 should be 1,2,3,4; L2 your cumulative time column)

20. Find the regression line

21. Sketch a graph, including the data points and the regression line. (Don't forget to draw a scale on both axes)



22. From the graph, how long would it take for the entire class to run our relay?

23. Sketch the graph.

24. What is the slope of your line?

25. What is the y-intercept of your line?

26. From your line, how long will it take for 25 Olympians to each run 400 meters?

Graphing Goes Live

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NCTM STANDARDS: Algebra, Geometry, Communication, Representation

GOALS:

Students will visually demonstrate their knowledge of graphing functions and associated concepts, varying in difficulty dependent upon the class involved. They will review the idea of coordinates, slope, y-intercept, derivative graphs, unit circle measurements and trigonometric graphs.

INTRODUCTION:

The creation of the Cartesian coordinate plane by René Descartes provided an important new concept to discover in the world of mathematics. Bonaventura Cavalieri followed suit soon after with the discovery of polar coordinates, making it easier to represent a function whose radius depends on an angle. With the infinite number of functions that we investigate in the mathematics classroom today, using the outdoors to find a new, exciting way to explain graphing to students can really help them visualize the image and characteristics of different graphs.

ACTIVITIES:

Part 1. Introduction: Teacher reviews how to construct graphs, and depending on subject matter, reintroduces the vocabulary associated with that activity.

Part 2. Student Construction: Students will use their knowledge of graphs to create three dimensional living graphs, with themselves as the coordinates. There will be three extensions below for the different subjects that can be explored.

ASSESSMENT:

At the end of the introduction activity, students will be taken to a previously created coordinate plane, Cartesian or polar, depending on the lesson. They will use their knowledge of slopes, y-intercepts, radiuses, and angles to represent those graphs. They will have to use their knowledge of these concepts in order to complete this activity successfully.

Teacher Notes – Cartesian plane, linear functions

Introduction – Students can often struggle with the abstract concepts of slope and y-intercept. Actually creating a three dimensional graph where they have to walk to create the functions can help them understand the ideas of "rise over run", or "where y=0". This real life example can help them to physically create the graph so they will visually see what you mean by these sayings. We represent linear functions in the form y= mx +b where m is the slope, b is the y-intercept, and –b/m is the x-intercept.

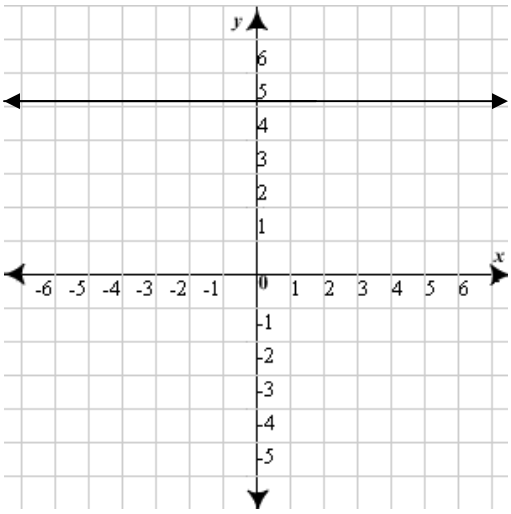
Preparation -
Creating the Coordinate Plane: Using your school parking lot if you have perpendicular spaces, you can easily create a coordinate plane using the middle line as the origin. If a school parking lot is not available, try to find a cement space where you can lay down masking tape to create the graphing plane. I suggest laying down one piece horizontally and then vertically, in alternating orders to assure you are making close to perpendicular and parallel lines. Also, try to make sure that the squares that you are creating are at least a foot apart, so the students will not be too close to each other when creating the graphs.

Choosing the Graphs: Because students will be learning about slope and y-intercept, it is important that there are a variety of slopes and y-intercepts explored during the activity. There should be negative and positive slopes, as well as varying y-intercept amounts. You could also challenge the students to find both x and y intercepts of the graphs if it exists.

Student Participation: Have a group of students create each group, and use as many "ordered pairs" (students) as you'd like to create the line. After creating the line, have another student walk the rise over run to determine the slope. You can vary this by giving students the Y-intercept and slope of the line and having them walk the rise over run from each point, in each direction. Clearly, this needs to be done until an X and Y intercept can be found, and the slope can be determined. Have students hold hands to actually create the line.

Solutions for the Guided Practice –

1. y= 5



X-Intercept: N/A

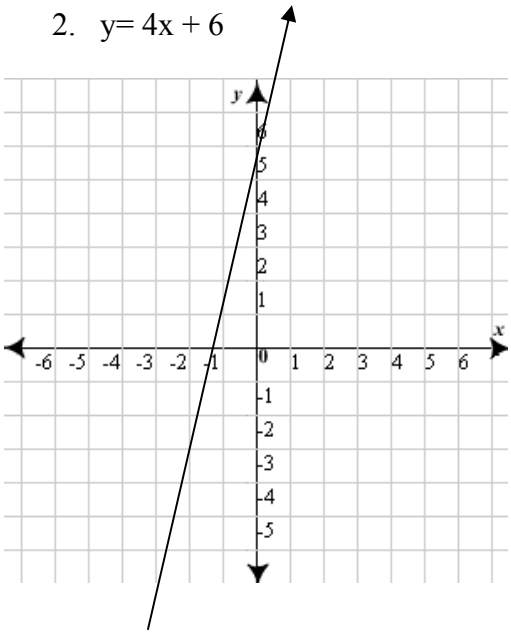
Y-Intercept: (0,5)

Slope: 0

The ordered pairs we used to create the graph:

X	F(X)
-3	5
-1	5
0	5
2	5
4	5

2. y= 4x + 6



X-Intercept: (-3/2,0)

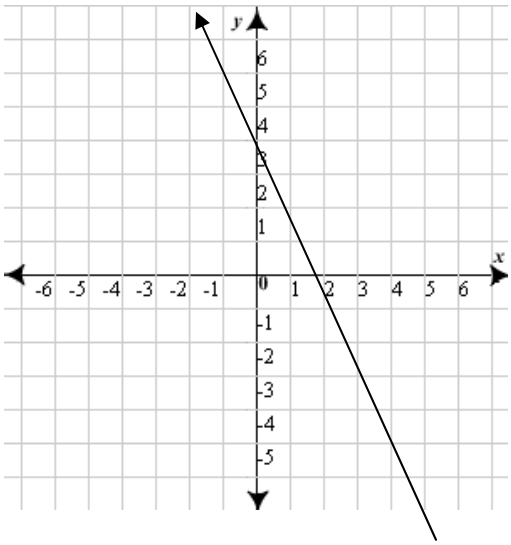
Y-Intercept: (0,6)

Slope: 4/1

The ordered pairs we used to create the graph:

X	F(X)
0	6
-1	2
0	3/2
-2	-2

3. y= -2x + 4



X-Intercept: (2,0)

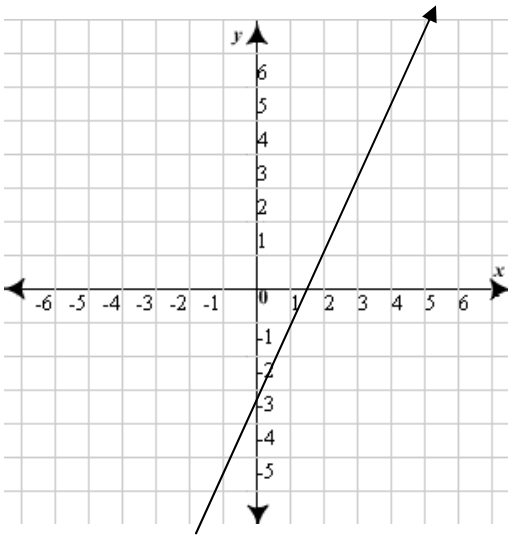
Y-Intercept: (0,4)

Slope: -2/1

The ordered pairs we used to create the graph:

X	F(X)
3	-2
2	0
0	4
-1	6

4. y= 5/2x - 7/2



X-Intercept: (7/5,0)

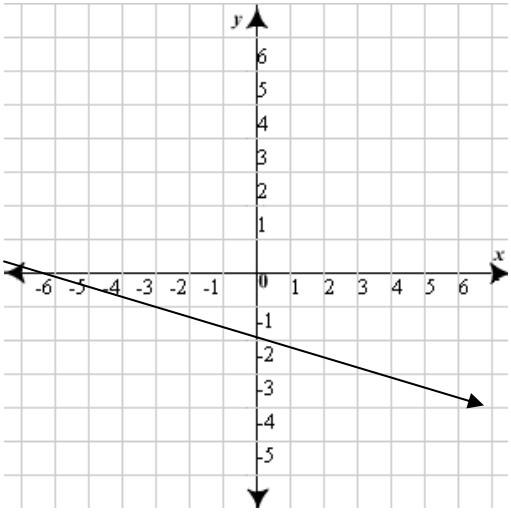
Y-Intercept: (0,-7/2)

Slope: 5/2

The ordered pairs we used to create the graph:

X	F(X)
-1	-6
1	-1
0	7/5
3	4

5. y= (-1/3)x -2



X-Intercept: (-6,0)

Y-Intercept: (0,-2)

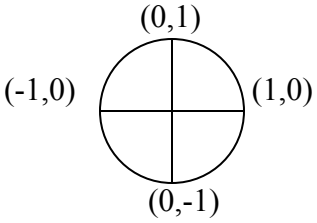
Slope: -1/3

The ordered pairs we used to create the graph:

X	F(X)
-6	0
-3	-1
0	-2
3	-3

Teacher Notes – Trigonometric Functions

Introduction – Students often have trouble understanding the curvature and characteristics of trigonometric functions, especially when the inside function is anything other than X. It is important for them to see the trigonometric functions in different forms so they know how the graph is manipulated when it changes. This activity will help students form these graphs in a fun and creative way. Make sure students understand the concept of the unit circle before participating in this activity as a good introduction to the topic.



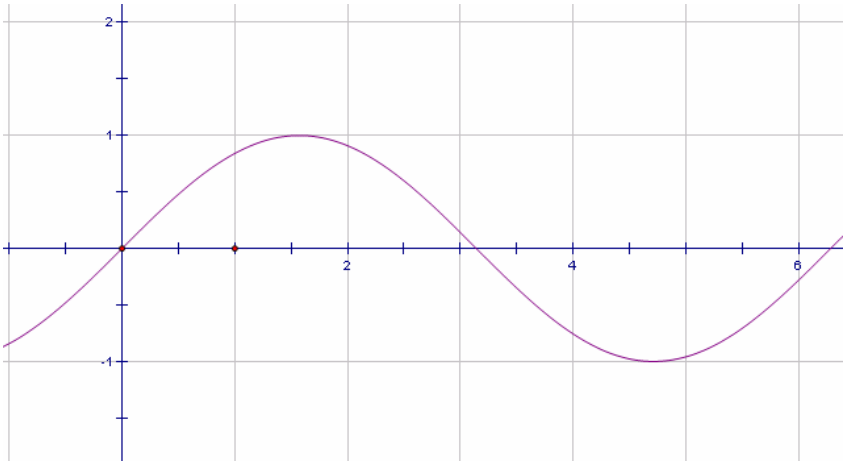
Preparation -
Creating the Coordinate Plane: Using your school parking lot if you have perpendicular spaces, you can easily create a coordinate plane using the middle line as the origin. If a school parking lot is not available, try to find a cement space where you can lay down masking tape to create the graphing plane. I suggest laying down one piece horizontally and then vertically, in alternating orders to assure you are making close to perpendicular and parallel lines. Also, try to make sure that the squares that you are creating are at least a foot apart, so the students will not be too close to each other when creating the graphs. Note that for this activity it will be best if you go ahead and mark out increments of π beforehand.

Choosing the Graphs: Begin using a simple trigonometric function, such as sin(x), and varying the graph to sin(2x). Do the same for cosine functions as well. Have students graph out what they think the new graphs will look like before you help them graph it on your human-sized coordinate plane.

Student Participation: Have a group of students create each graph, and use as many "ordered pairs" (students) as you'd like to create the function. Because there is curvature in the graph, it will be necessary to use as many students as possible. It is best if you give coordinates for students to stand on, and then after having called out as many coordinates as needed, have them connect by holding hands to form the graph. After creating the function, have another student walk the length of the graph to determine when it begins repeating itself to determine the period. You can vary this by giving students the period, range, radius, and a few points and tell them to figure out the function.

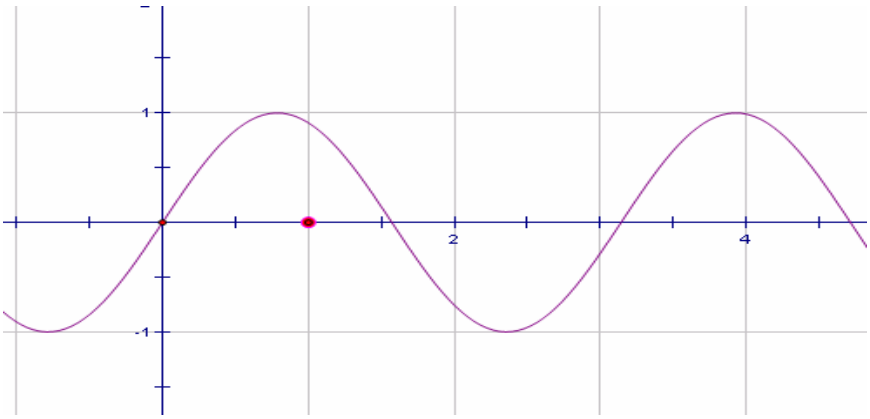
Solutions for the Guided Practice –

1. y= sin(X)



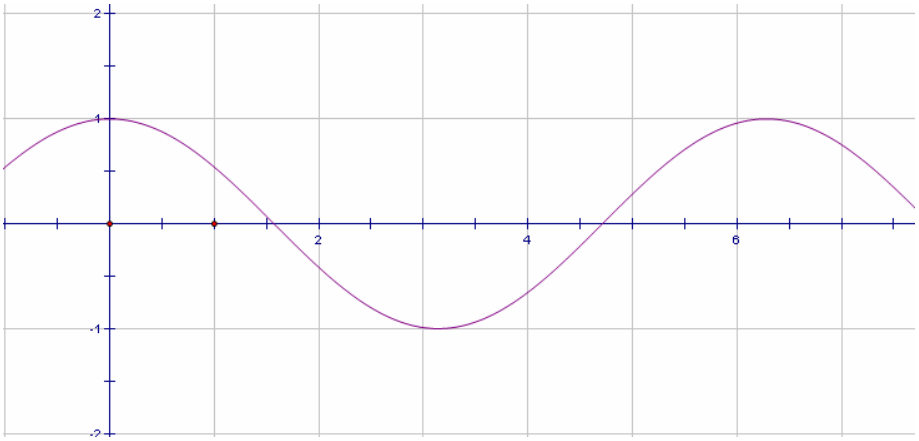
Minimum: -1
Maximum: 1
Range: $-1<y<1$
Period: 2π

2. y= sin(2X)



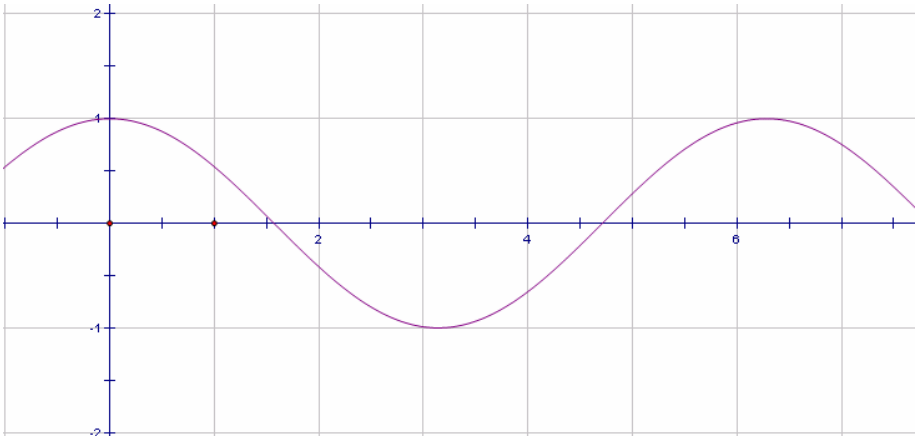
Minimum: -1
Maximum: 1
Range: $-1<y<1$
Period: π

3. y= cos(X)



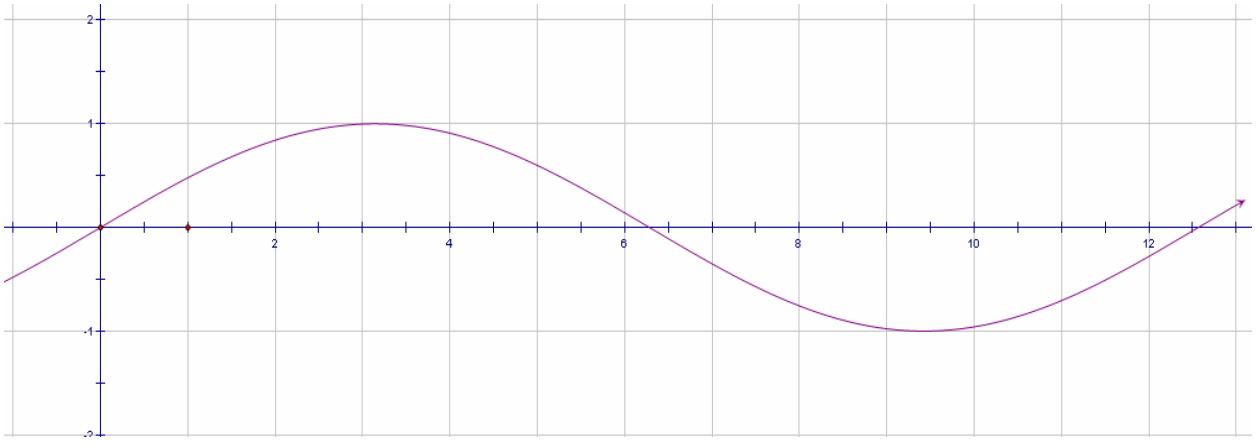
Minimum: -1
Maximum: 1
Range: $-1<y<1$
Period: 2π

4. y= cos(3X)



Minimum: -1
Maximum: 1
Range:-1<y<1
Period: $2\pi/3$

5. y= sin(1/2X)



Minimum: -1
Maximum: 1
Range:-1<y<1
Period: 4π

Teachers Notes – Polar Coordinates

Introduction – Understanding that there is more than one plane system can often baffle students at the beginning. Learning about polar coordinates is a vital part to upper level mathematics, giving insight to functions in a new setting. Having the knowledge to go back and forth between both Cartesian and polar coordinate planes is a great asset to any student. That conversion is found in the following equations:

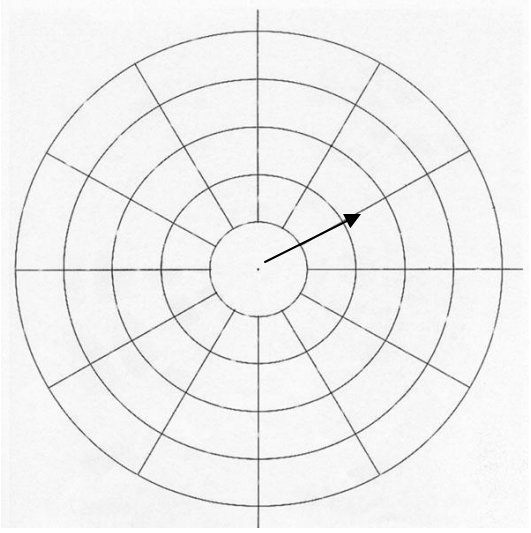
$X = r \cos \theta$
$Y = r \sin \theta$
$R^2 = \sqrt{(x^2+y^2)}$

Preparation:
Creating the Plane: It's not as hard to create a polar plane as one might think. Using masking tape, create an origin that is a set of perpendicular lines. Using a protractor, measure off increments of degrees you will be using around the top half of the graph. I suggest 30°, 60°, 90°, 120°, 150°, and 180°. Once you have marked out those pie pieces, use masking tape to make a radius to and through your origin to the other side of the plane. This way when you create a 30° wedge, you are also creating the 210° at the same time. You will half the work that you actually have to do by this process. Now you should have what appears to be many wedges. To complete the plane, take some sidewalk chalk and draw unit circles around the whole figure as evenly as possible. This completes your polar coordinate plane.

Choosing the Graphs: With polar coordinates there are many ornate graphs that can be represented. Because you are outside and it will be hard to check for accuracy, it is best to stick to simpler graphs as a way to introduce the subject, rather than try to draw an intricate polar rose outdoors, although it can be done.

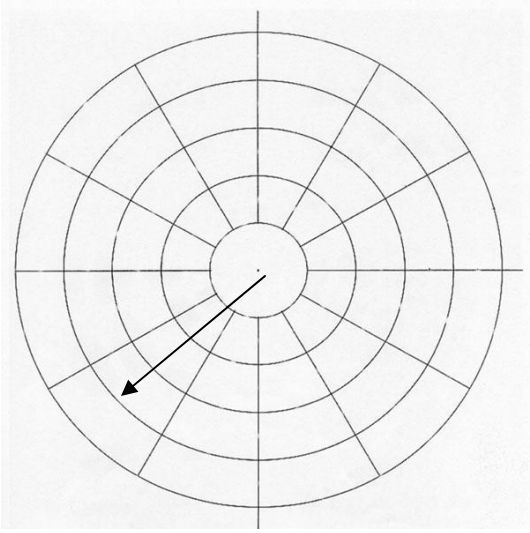
Solutions to the Guided Practice –

1. $r = 1; \theta = \pi/6$



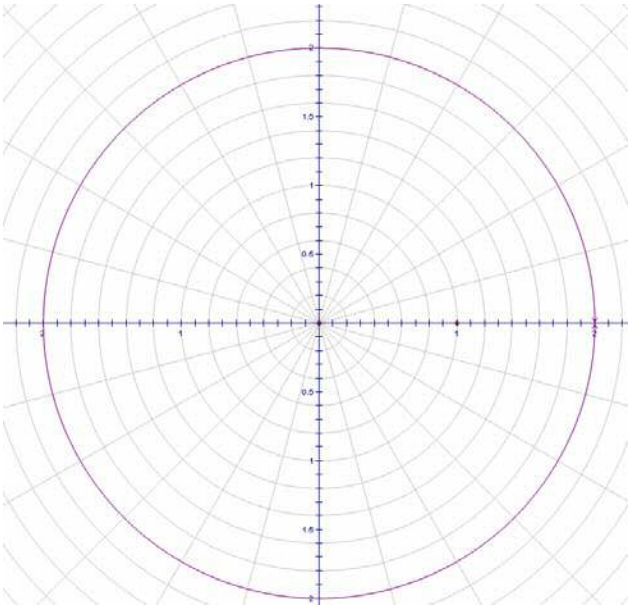
Radius: 1
Theta: $\pi/6$

2. $r = 3$; $\theta = 5\pi/4$



Radius: 3
Theta: $5\pi/4$

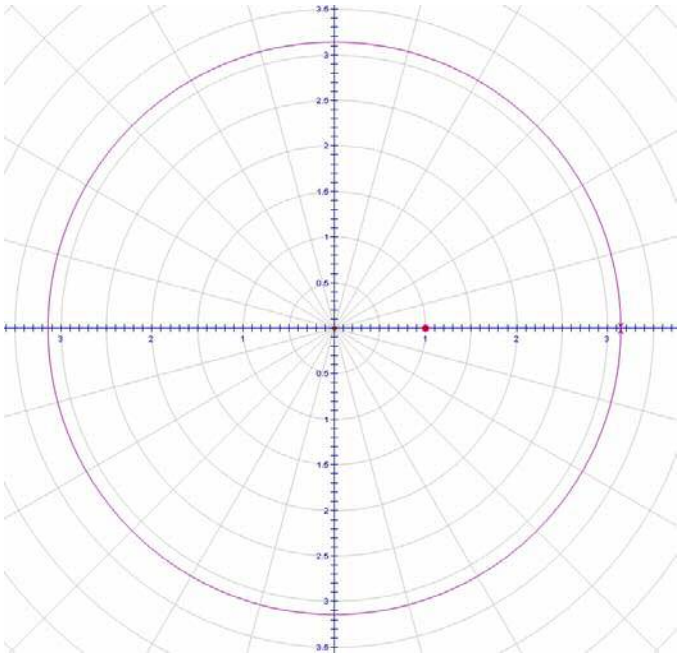
3. $r(\theta) = 2$



Radius: 2
Theta: 2π

R	θ
2	2π
2	$\pi/6$
2	$3\pi/2$
2	$5\pi/4$
2	$10\pi/6$

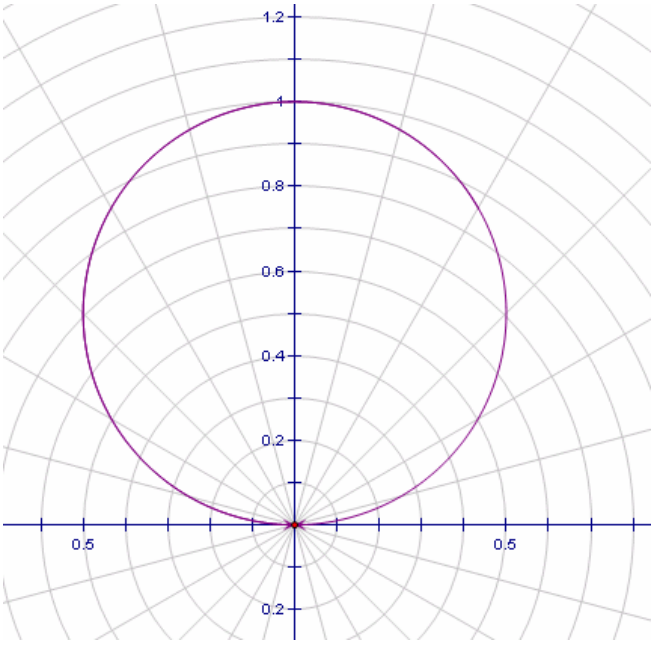
4. $r(\theta) = \pi$



Radius: π
Theta: 2π

R	θ
π	2π
π	π
π	4π/6
π	8π/3
π	7π/4

5. $r(\theta) = \sin(\theta)$



Radius: 1/2
Theta: 2 π

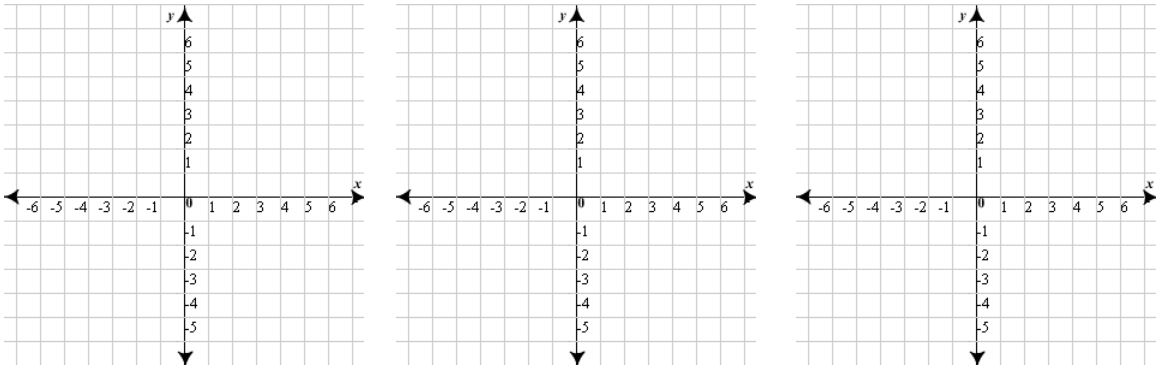
R	θ
0	0
1	π/2
.5	π/6
.7	π/4
.5	5π/6
.7	3π/4

Graphing Goes Live – Linear Functions

Graphs are a visual way to represent functions. Using the Cartesian coordinate plane, we can describe each function by its individual characteristics. Within each linear function, there is a slope and an intercept. Your job is to investigate each graph and using people as ordered pairs, create the representation in the Cartesian plane that correctly symbolizes each function.

Before creating each graph with other students in your class, graph what you think it will look like below. Then, graph what it actually looks like. Lastly, fill in the table and blanks for each function.

6. y= 8

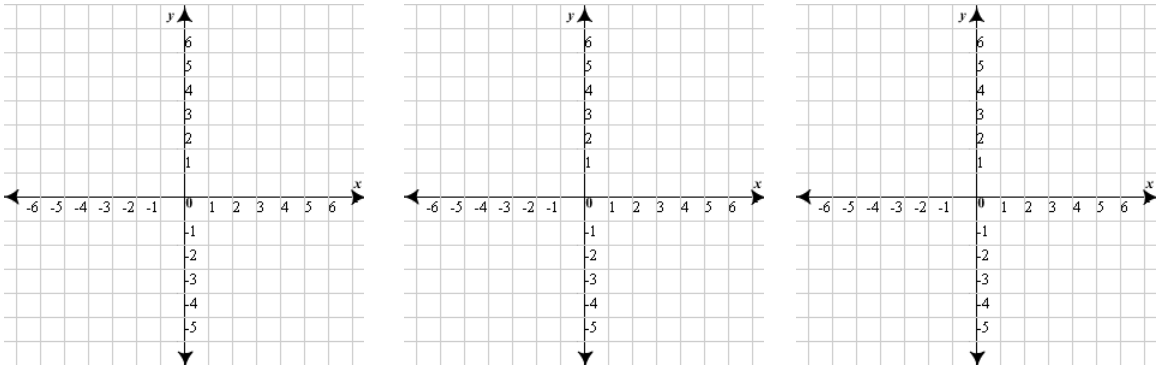


X-Intercept: _____ Y-Intercept _____ Slope:

The ordered pairs we used to create the graph:

X	F(X)

7. y= 4x + 6

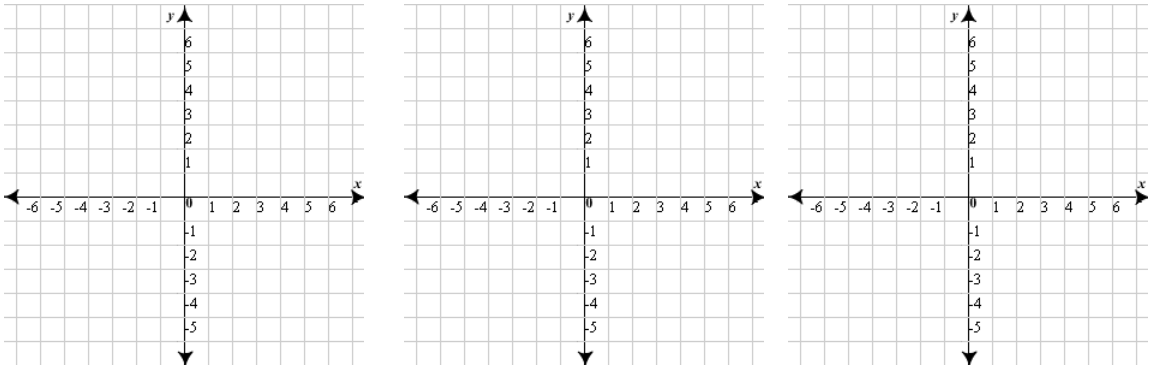


X-Intercept: _____ Y-Intercept _____ Slope:

The ordered pairs we used to create the graph:

X	F(X)

8. y= -2x + 4

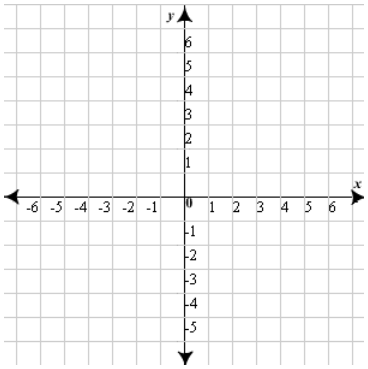
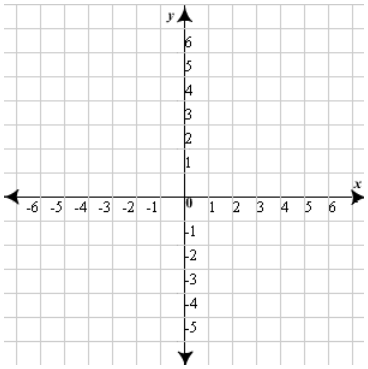
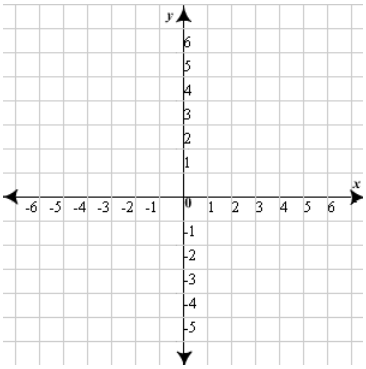


X-Intercept: _____ Y-Intercept _____ Slope:

The ordered pairs we used to create the graph:

X	F(X)

9. y= 5/2x - 7/2

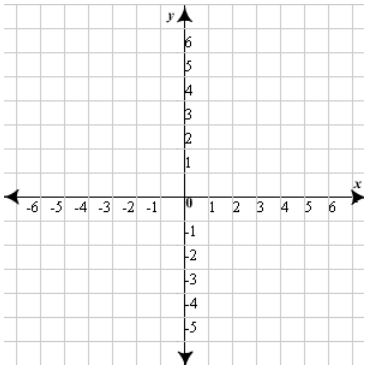
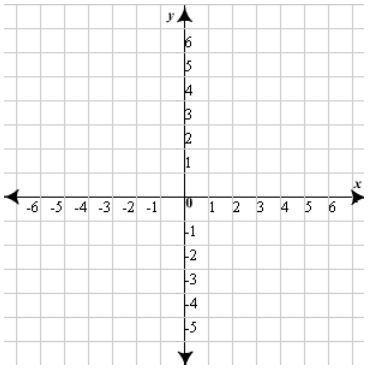
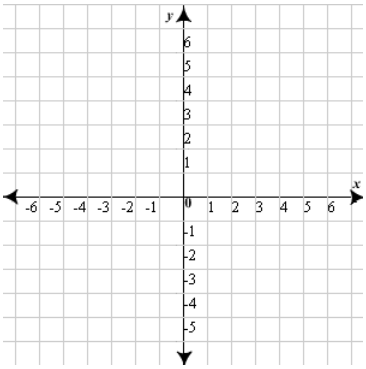




X-Intercept: _____ Y-Intercept _____ Slope:

The ordered pairs we used to create the graph:

X	F(X)

10. y= (-1/3)x -2

X-Intercept: _____ Y-Intercept _____ Slope:

The ordered pairs we used to create the graph:

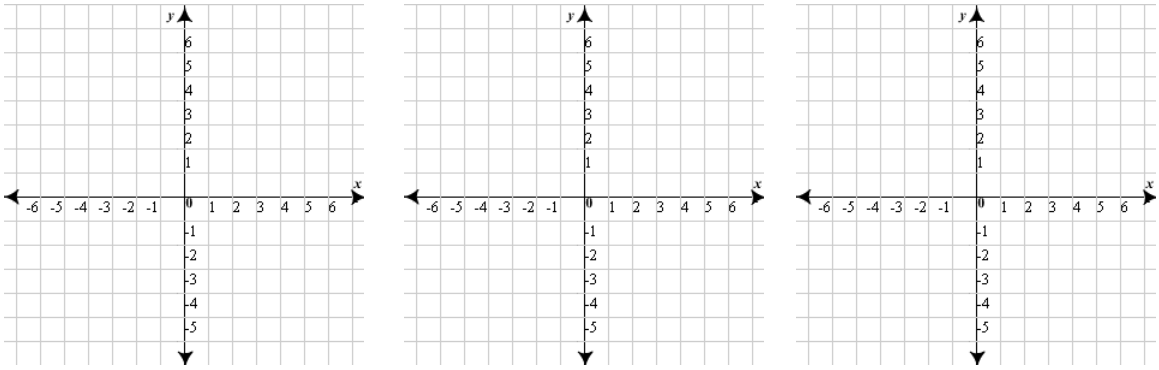
X	F(X)

Graphing Goes Live – Trigonometric Functions

Graphs are a visual way to represent functions. Using the Cartesian plane, we can plot out the graphs of trigonometric functions to see their individual characteristics.

Before creating each graph with other students in your class, graph what you think it will look like below. Then, graph what it actually looks like. Lastly, fill in the blanks for each function.

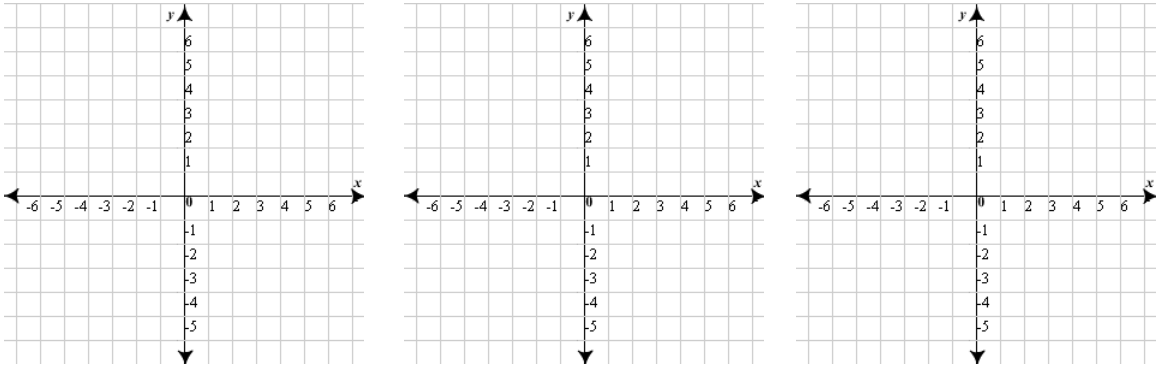
6. y= sin(X)



Minimum: Maximum:

Range: Period: Radius:

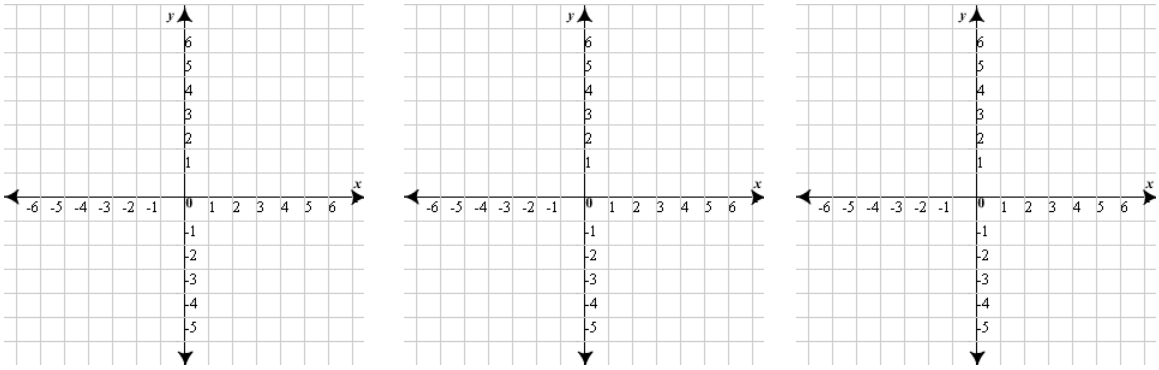
7. y = sin(2X)



Minimum: Maximum:

Range: Period: Radius:

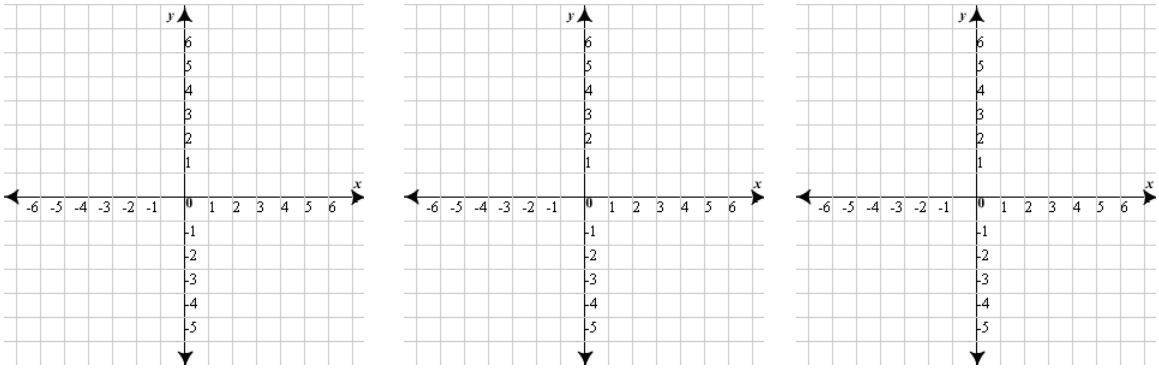
8. y= cos(X)



Minimum: Maximum:

Range: Period: Radius:

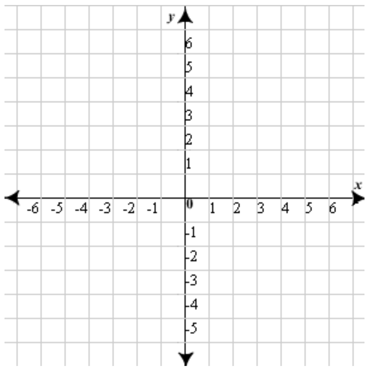
9. y= cos(3X)

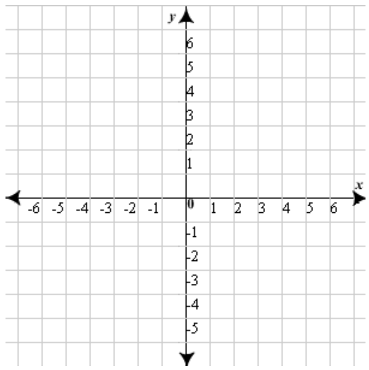


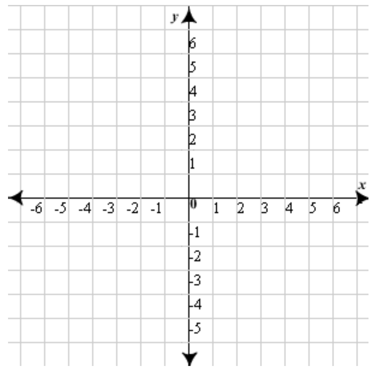
Minimum: Maximum:

Range: Period: Radius:

10. y= sin(1/2X)







Minimum: Maximum:

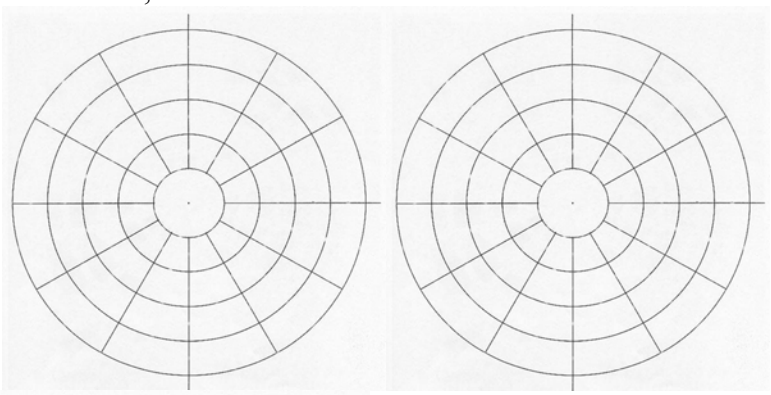
Range: Period: Radius:

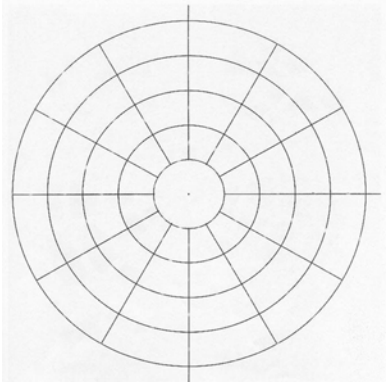
Graphing Goes Live – Polar Coordinates

Graphs are a visual way to represent functions. Using the Polar coordinate plane, we can plot out the graphs of different functions to see their individual characteristics.

Before creating each graph with other students in your class, graph what you think it will look like below. Then, graph what it actually looks like. Lastly, fill in the blanks for each function.

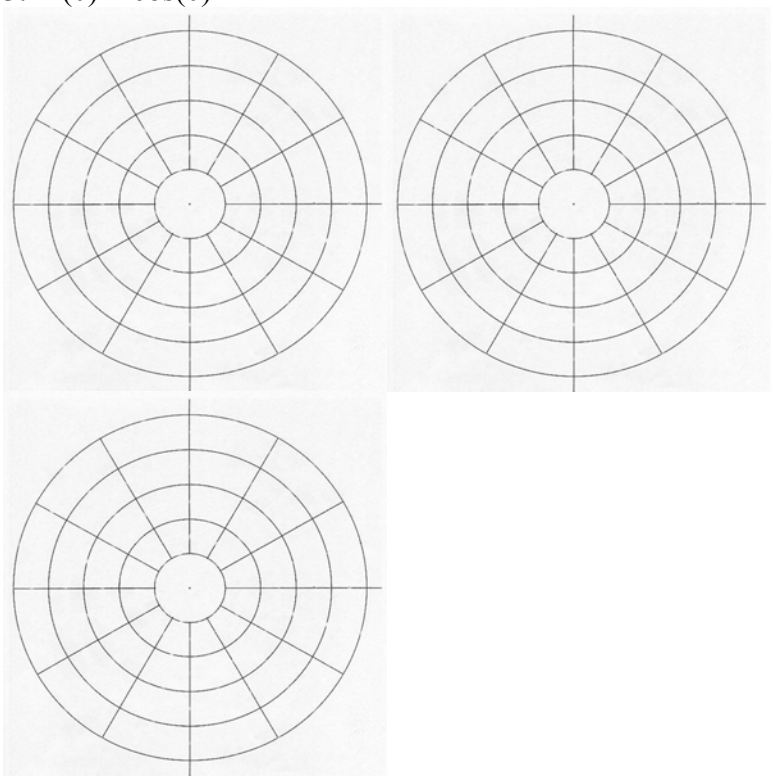
2. $r = 1; \theta = \pi/6$





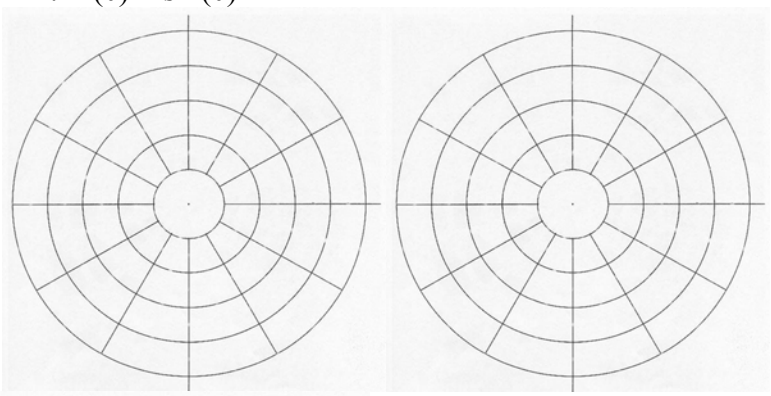
Radius:
Theta:

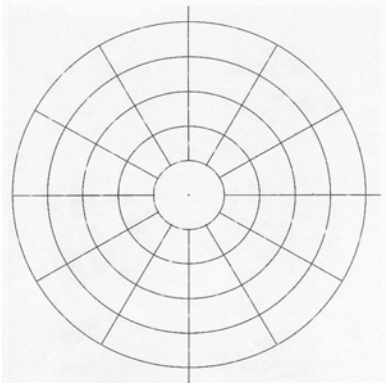
3. $r(\theta) = \cos(\theta)$



Radius:
Theta:

4. $r(\theta) = \sin(\theta)$

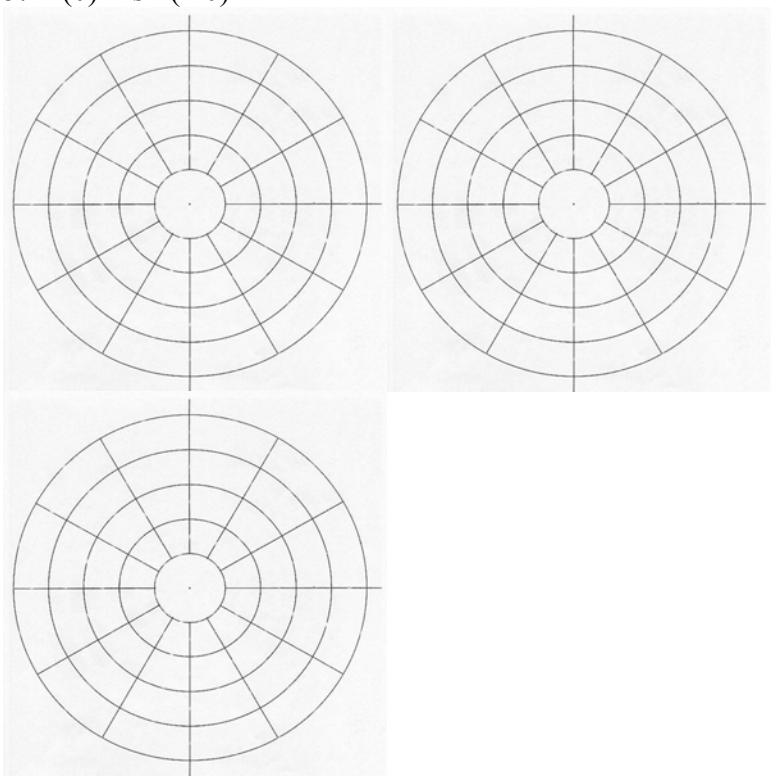




Radius:
Theta:

R	θ

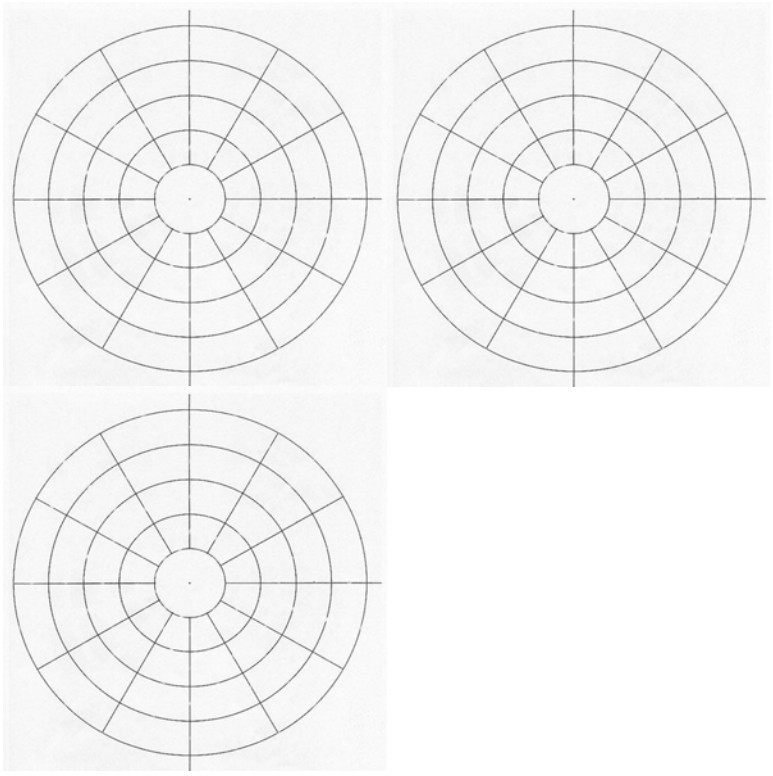
5. $r(\theta) = \sin(2\theta)$



Radius:
Theta:

R	θ

6. $r(\theta) = 2 \sin (2 \theta)$



Radius:
Theta:

R	θ

Pulse Rates and Fitness

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INTRODUCTION: It is important for young people to understand the effect fitness level has on their bodily systems. Pulse rate is often used as a measure of whether or not a person is in good physical condition. There are two types of pulse rates: resting and exercise. A resting pulse rate occurs when a person's heart is at rest and is best measured in the morning, after a good night's sleep, or before you get out of bed. The heart beats about 60 to 80 times a minute when at rest. Resting heart rate usually rises with age, and it's generally lower in physically fit people. The exercise pulse rate occurs when a person is involved in physical activity.

NCTM STANDARDS: Measurement, Data Analysis & Probability, Communication, Connections, Representation

MATERIALS: jump ropes, stopwatch, graphing calculator

GOALS: Students will measure and record resting and exercise pulse rates. They will create box plots to represent class data and to reflect on the effect of fitness level on pulse rate.

ACTIVITIES:

Part I: Introduction

Teacher will demonstrate how to find a pulse rate using either the carotid artery on the side of the neck or the radial pulse on the wrist. The teacher will also review how to construct box and whisker plots using the calculator as well as the meanings of outliers, extremes, median, mean, and quartiles.

Part 2: Measure Pulse Rates

In groups, students will first have his/her pulse rate recorded. Then they will jump rope for two minutes and then have the exercise pulse rate recorded. Finally, after one minute of rest, they will record the pulse rate again.

Part 3: Compile Data and Construct Box Plots

After recording the pulse rates, students will go to the board and put the pulse rates of their group on the board. Once the class' data is compiled, students will put the sets of data in their calculators and create box plots or construct them by hand.

ASSESSMENT: Students will answer questions based on the box plots created. Students will use their knowledge of box plots and measures of central tendency to answer questions about the effects of fitness level on pulse rate.

Part I: Introduction

Teacher will demonstrate how to find a pulse rate using either the carotid artery on the side of the neck or the radial pulse on the wrist. To find a pulse using the carotid artery, place two fingers underneath the jaw-line near the ear. To find a pulse using the radial pulse, place two fingers on the bottom of the wrist near the palm of the hand. Many students will have trouble finding a pulse using the wrist, so encourage your class to use the carotid artery.

Pulse rate is measured in beats per minute (bpm). When measuring resting pulse rate, students can count the number of beats for 15 seconds and then multiply by 4 to get the rate. However, when measuring the exercise pulse rate, make sure and remind students to count beats for the full 60 seconds since the number of beats will slow as students cool down and time passes after activity.

In order to construct a box plot, students must complete the following steps:

- Enter their data into a list by going to STAT then Edit.
- Then, they enter their data into L1.
- To get out, press 2^{nd} MODE.
- Next, they press 2^{nd} "Y=" to take them to the STAT PLOTS screen.
- At "1:" press ENTER. Highlight "On" and press ENTER.
- Use the arrow keys to select one of the two box plot types (and press ENTER).
- The difference between the two is that the first one shows outliers.
- Finally, press ZOOM then 9:ZoomStat. The box plot should appear.

On the box plot, there will be 5 key components presented:

minX is the smallest valued data entry
maxX is the largest valued data entry
Q1 is the lower quartile (25^{th} percentile)
Q3 is the upper quartile (75^{th} percentile)
Med is the median (middle) value

Other noteworthy vocabulary are:

Outliers: Any data observation which lies more than 1.5*IQR lower than the first quartile or 1.5*IQR higher than the third quartile
Extremes: Any data observation which lies more than three times the IQR to the left and right from the first and third quartiles
Mean: the average of the data set

Part II: Measure Pulse Rates

Have students pair up and give each group one jump rope and one stop watch and follow these instructions:

1) Relax for a minute or two.
2) Working with your partner, find your resting heart rate by counting the beats per minute for 15 seconds and multiplying this number by 4.
3) Jump rope for 2 minutes
4) Find heart rate immediately following exercise and record on paper
5) Rest for 1 minute recovery period (while your partner is recording exercise heart rate)
6) Find heart rate after 1 minute recovery period and record
7) Subtract the 1 minute recovery period pulse rate from the exercise pulse rate and record. We'll call this the post-exercise pulse rate drop

Part III: Compile Data and Construct Box Plots

After each group has completed the activity, they should go to the board and anonymously record the resting pulse rate, the exercise pulse rate, and the post-exercise pulse rate drop of both group members on a chart on the board. This chart can be a stem and leaf plot or can simply be numbers listed for each type.

After compiling the data, the students must create box plots for each of the three data sets using their calculators. The teacher will also construct the three box plots on a calculator and project the image onto the overhead or TV screen.

Part IV: Assessment

Students will be asked to get back into their groups for a think-pair-share activity. In their groups, they will interpret the graphs they've made. Be sure to remind students that they are not only comparing centers, but also spreads and shapes of the graphs.

Students should also specifically discuss the box plot for recovery heart rate. Using the scale below, what percentage of the class is in excellent shape? In average shape? In poor shape?

Excellent: drop of 50 beats or more
Average: drop of 30-45 beats
Poor: drop of fewer than 30 beats

Then, they are to write a journal entry answering these two questions:
1) How would you describe the average fitness level of the class based on the results? Do you think people who consider themselves athletes have better scores than those who do not?
2) How could a physician or a personal trainer use your results to determine whether a client is in good physical condition?

Extension Questions

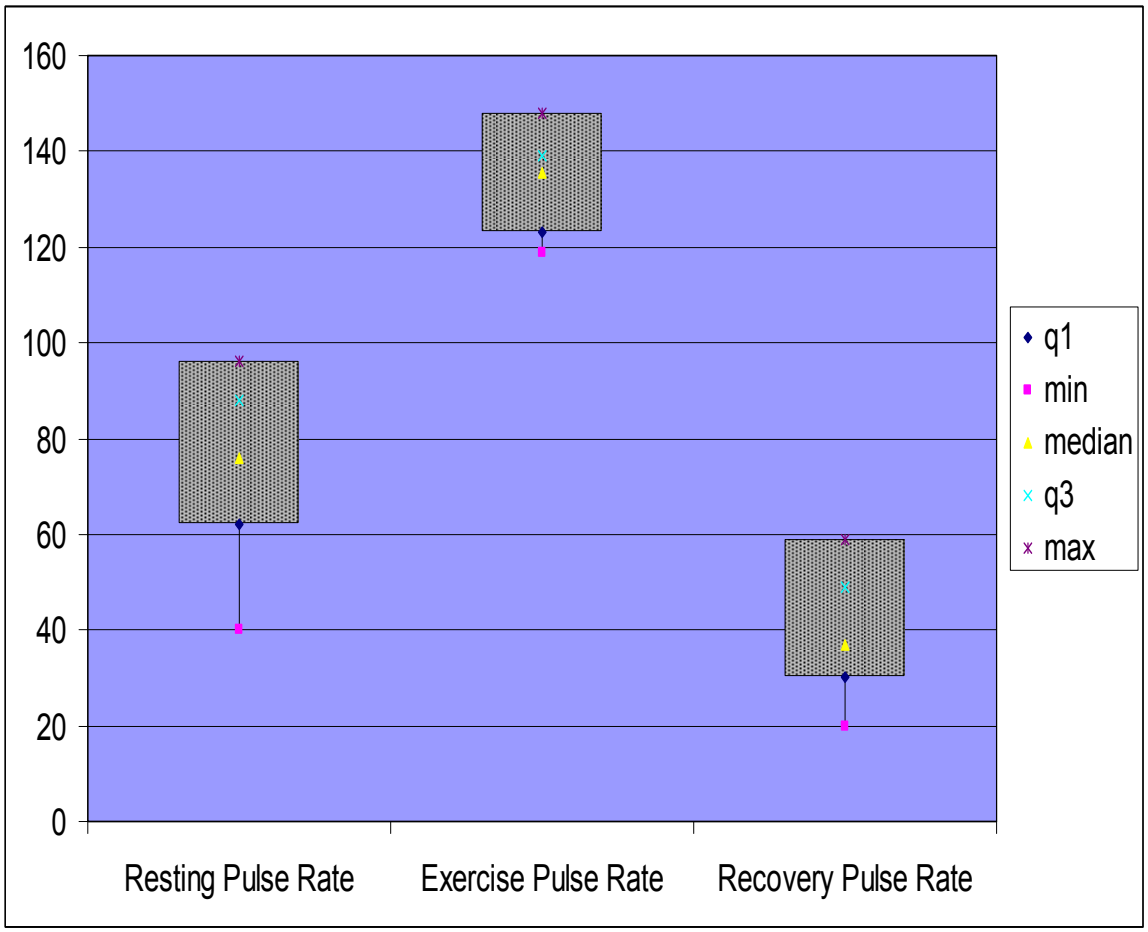
1) With any form of measurement, there is always error. In looking at our methods for measuring pulse rate, where could error have occurred? What could we do to limit error?

2) Maximum heart rate is the maximum heart rate that a person should achieve during maximal physical exertion. Also, target heart rate takes your fitness level into account and prescribes a range that your heart rate should fall into during exercise. If maximum heart rate is found by subtracting your age from 220 and your target heart rate is 50-85% of your maximum heart rate, what is your target heart rate? How would you use this information when designing a fitness plan for yourself?

Sample Data Set and Solution

Student	Resting Pulse Rate	Exercise Pulse Rate	Recovery Pulse Rate
A	96	148	20
B	94	147	24
C	92	145	25
D	90	143	26
E	89	141	27
F	88	139	30
G	88	139	30
H	86	138	30
I	85	138	32
J	84	137	32
K	82	137	33
L	80	136	33
M	77	136	35
N	75	135	39
O	73	134	44
P	70	133	46
Q	69	133	46
R	68	129	47
S	65	125	48
T	62	123	49
U	58	122	50
V	55	121	52
W	50	121	55
X	48	120	56
Y	45	120	57
Z	40	119	59

Statistic	Resting Pulse Rate	Exercise Pulse Rate	Recovery Pulse Rate
q1	62	123	30
min	40	119	20
median	76	135.5	37
q3	88	139	49
max	96	148	59



For information on how to graph box plots in Excel, visit:
http://www.mis.coventry.ac.uk/~nhunt/boxplot.htm

Choosing a GREEN Car

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INTRODUCTION: Environmental issues are important to everyone living on the earth today. Global warming, air pollution, and use of non-renewable resources are all sources of concern. This project uses online government information to compare cars on the following variables: annual petroleum consumption, greenhouse gas emissions, and air pollution score. Greenhouse gases refer to emissions that cause global climate change. Air pollution refers to vehicle emissions that are harmful to human health and/or cause smog.

NCTM STANDARDS: Data Analysis & Probability, Communication, Connections, Representation, Problem Solving

MATERIALS: Computers with Internet connection and spreadsheet software

GOALS: Students will collect online data, represent it in a table and a graph, and analyze outcomes to draw conclusions.

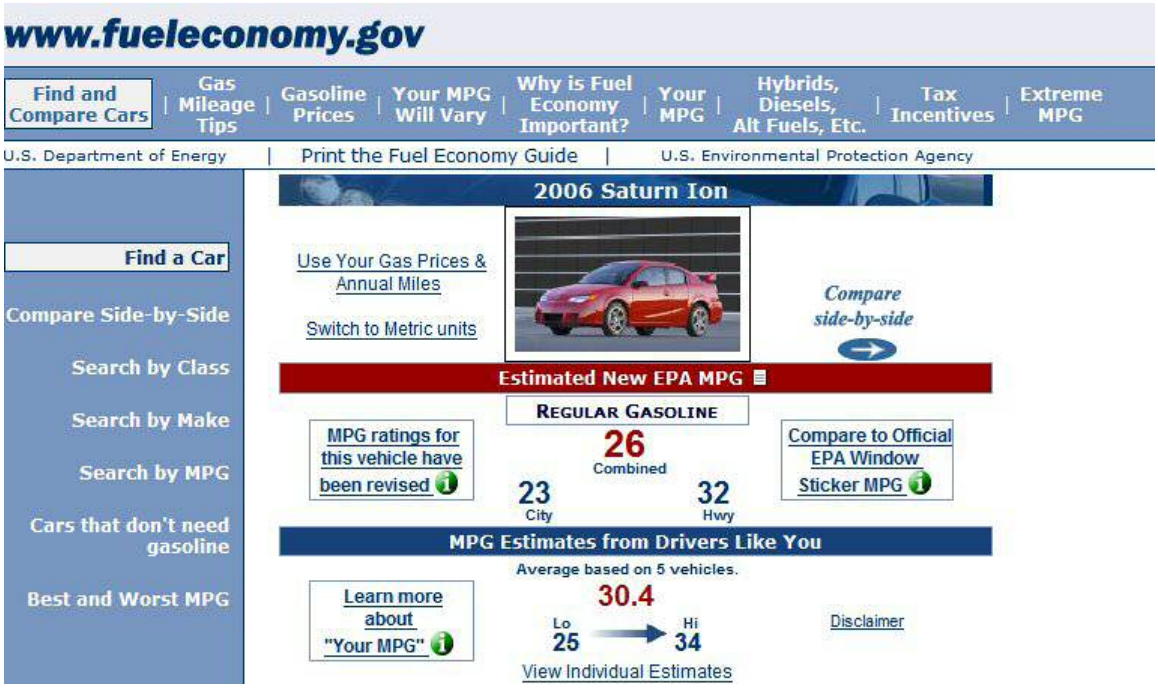
ACTIVITIES:

- Working in small groups, students will select five different cars to research.
- Using the <http://www.fueleconomy.gov> website, they will find each car and record the annual petroleum consumption, greenhouse gas emissions, and air pollution score. See Worksheet.
- Students will record the data in a spreadsheet and construct a line graph to represent it visually.
- Each group will submit a report, including their table, graph, and conclusions about choice of car.

ASSESSMENT: Group reports will be graded using attached rubric. Each individual student will submit a reflection about the data-driven decision process and their thoughts about "going green."

Choosing a GREEN Car

1. **Go to: http://www.fueleconomy.gov**
2. **Click on "Find and Compare Cars" or "Hybrids, etc."**
3. **Select a "Model Year" and then a "Make"**
4. **Select a car. Click "Compare."**



5. **Record data in table.**

Choosing a GREEN Car

See http://www.fueleconomy.gov

Car	Annual Petroleum Consumption	Greenhouse Gas Emissions	Air Pollution Score

Notes:

Annual Petroleum Consumption indicates the number of barrels of petroleum used by the vehicle each year. This includes fuel burned by the vehicle as well as petroleum used in producing, refining and delivering it. Using fewer barrels is better.

The **Greenhouse Gas Emission** rating shows the annual amount of greenhouse gases emitted by the selected vehicle in relation to the lowest and highest amounts emitted by other vehicles. Greenhouse gases refer to emissions that cause global climate change. A lower score is better.

The **Air Pollution Score** represents the amount of health-damaging and smog-forming airborne pollutants the vehicle emits. Air pollution refers to vehicle emissions that are harmful to human health and/or cause smog. Scoring ranges from 0 (worst) to 10 (best). A higher score is better.

Choosing a GREEN Car – Scoring Rubric

	Excellent 5 4	Satisfactory 3 2	Below Expectations 1 0
Internet Research _____	Students effectively navigated Internet and easily gathered information from electronic database.	Students navigated Internet and gathered information from electronic database.	Students required assistance to navigate Internet and gathered information from electronic database.
Data Table _____	Data are presented accurately and neatly in table.	Data are presented mostly accurately and neatly in table.	Data table contains several inaccuracies and/or is not neat.
Graph _____	Graph is appropriate and accurately represents data. Graph includes title, legend, and labels.	Graph is appropriate and mostly accurately represents data. Graph includes title, legend, and labels.	Graph may not appropriately represent data and has inaccuracies and/or is missing titles, legend, or labels.
Analysis and Conclusions _____	Students carefully analyzed the information collected and drew appropriate conclusions supported by evidence. Mathematical reasoning was evident.	Students' conclusions could be supported by stronger evidence. Level of analysis could have been deeper. Mathematical reasoning was mostly adequate.	Students' conclusions simply involved restating information. Conclusions were not supported by evidence. Mathematical reasoning was not evident.
Individual Reflection _____	The reflection responds to the group process and to thoughts on "Going Green." The paper is well written and organized with complete sentences and no mechanics problems.	The reflection responds somewhat to the group process and to thoughts on "Going Green." The paper is well written and organized with complete sentences and few mechanics problems.	The reflection does not clearly respond to the group process and to thoughts on "Going Green." The paper is poorly written and not well organized with several mechanics problems.

Total_____

Comments:

Sample Solution:

Car	Annual Petroleum Consumption	Greenhouse Gas Emissions	Air Pollution Score
Accord	13.2	7.1	6
Taurus	17.1	9.2	3
Prius	7.4	4.0	8
Hummer	22.8	16.2	6
Viper	26.3	14.1	2